A Non-monotone Filter Algorithm

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Problem

The general nonlinear programming problem:

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & c_{\mathcal{E}}(x) = 0 \\ & c_{\mathcal{I}}(x) \geq 0, \end{array}$$

for $x \in \mathbb{R}^n$, f and c smooth.
(today: emphasis on inequality constraints)

Define the maximum constraint violation :

$$\theta(x) = \max[0, -c_1(x), \ldots, -c_m(x)] = \|c_-(x)\|_\infty$$

At the same time:

- reduce the objective function $f(x)$
- reduce constraint violation $\theta(x)$

$\Rightarrow$ CONFLICT

The Step Selection at x_k

- (approximate) SQP step $\Rightarrow$ s_k
  - satisfies linearized constraints (when possible)
    normal step $n_k \Rightarrow x_k + n_k$
  - reduces a model m_k of the Lagrangian
    tangential step $t_k \Rightarrow x_k + n_k + t_k$
- impose trust-region to ensure global convergence (Δ_k)

s_k approximately solves the TRQP(x_k, Δ_k) :

$$\begin{array}{ll} \text{minimize} & m_k(s) = f_k + g_k^T s + \frac{1}{2} s^T W_k s \\ \text{subject to} & c_k + A_k s \geq 0 \\ \text{and} & \|s\|_? \leq \Delta_k, \end{array}$$

(. . . but linesearch technique also possible!)

The filter point of view

Fletcher and Leyffer replace question:

what is a better point?

by:

What is a worse point?

Of course, y is worse than x when

$$f(x) \leq f(y) \text{ and } \theta(x) \leq \theta(y)$$

(y is dominated by x)

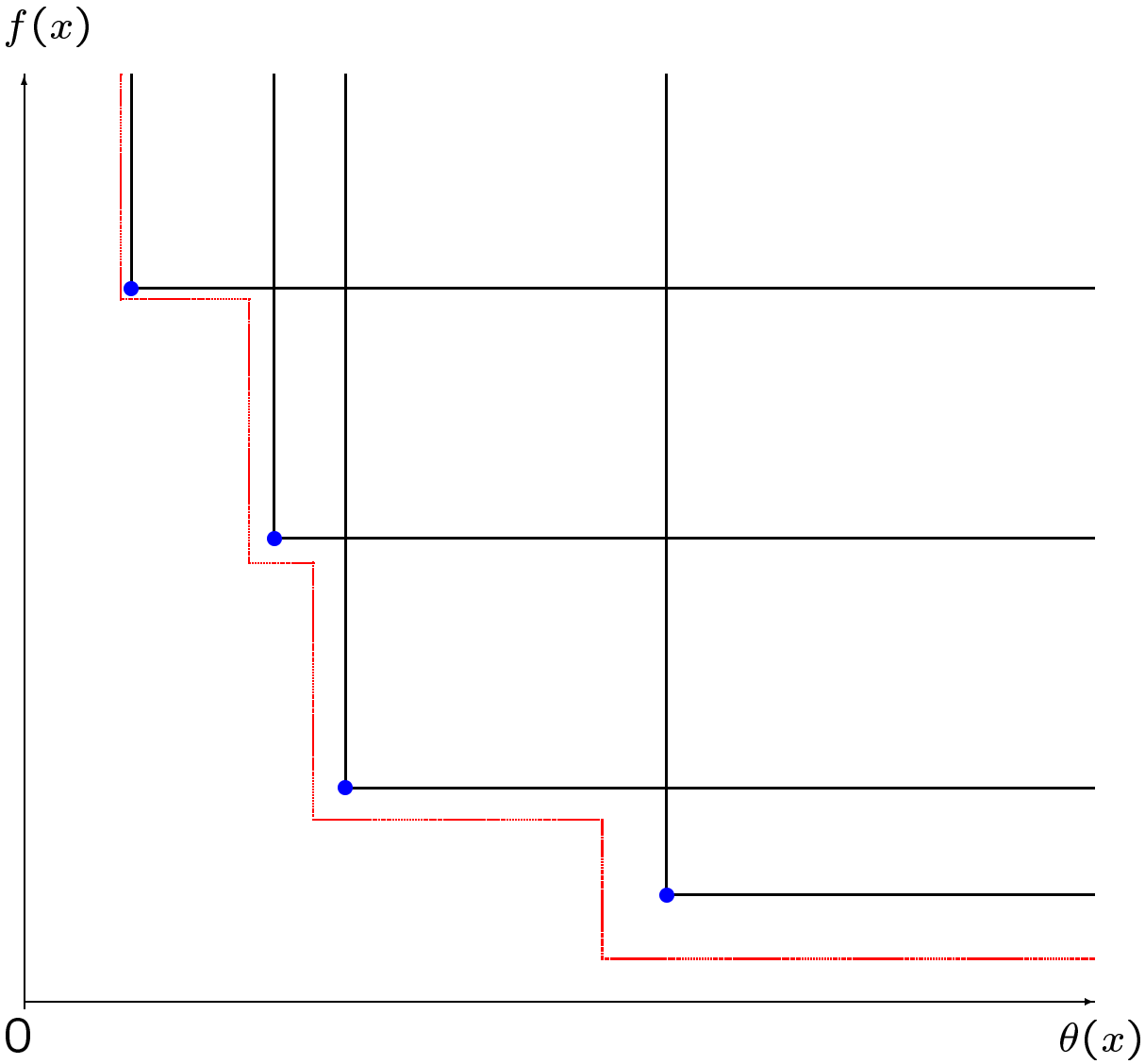
When is $x_k + s_k$ acceptable?

Potential problem: Maratos effect

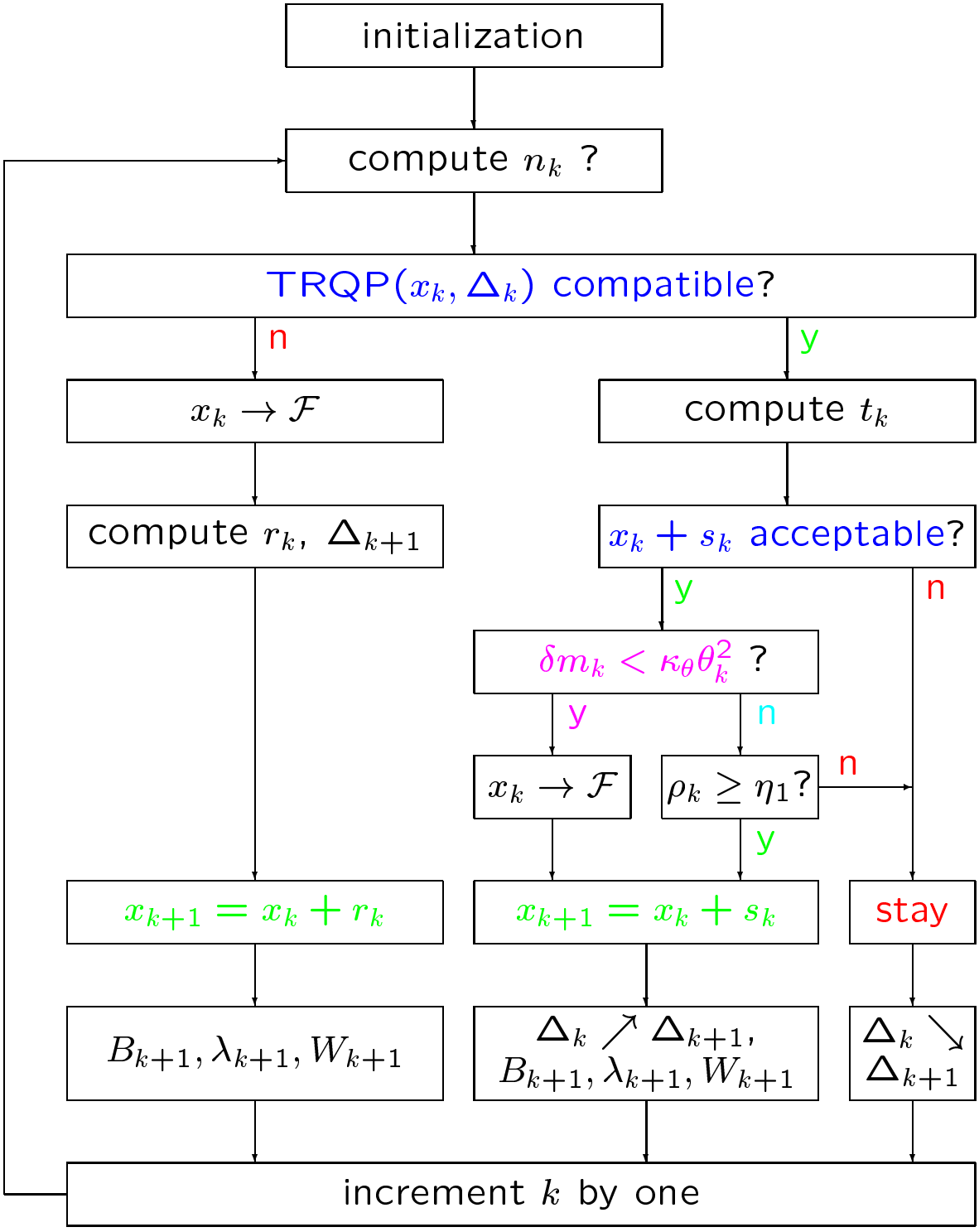
The "Standard" Filter

Idea: Build a numerical method where non-dominated points are accepted.

remember accepted points $\Rightarrow$ filter ($\mathcal{F}$)

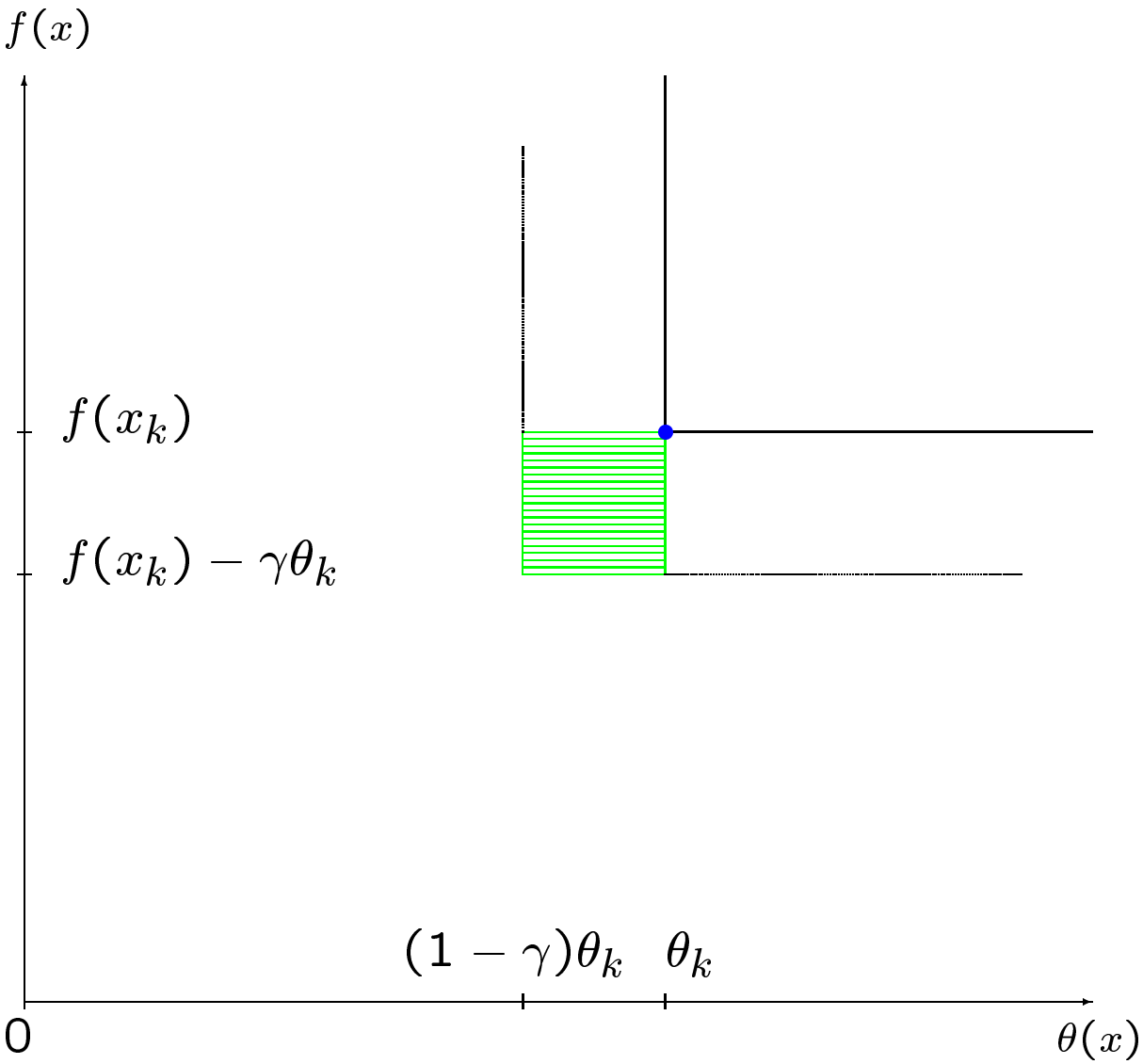


The Filter Algorithm



Filling up the filter (version 1)

Note: filter area is bounded in the (f, θ) space!

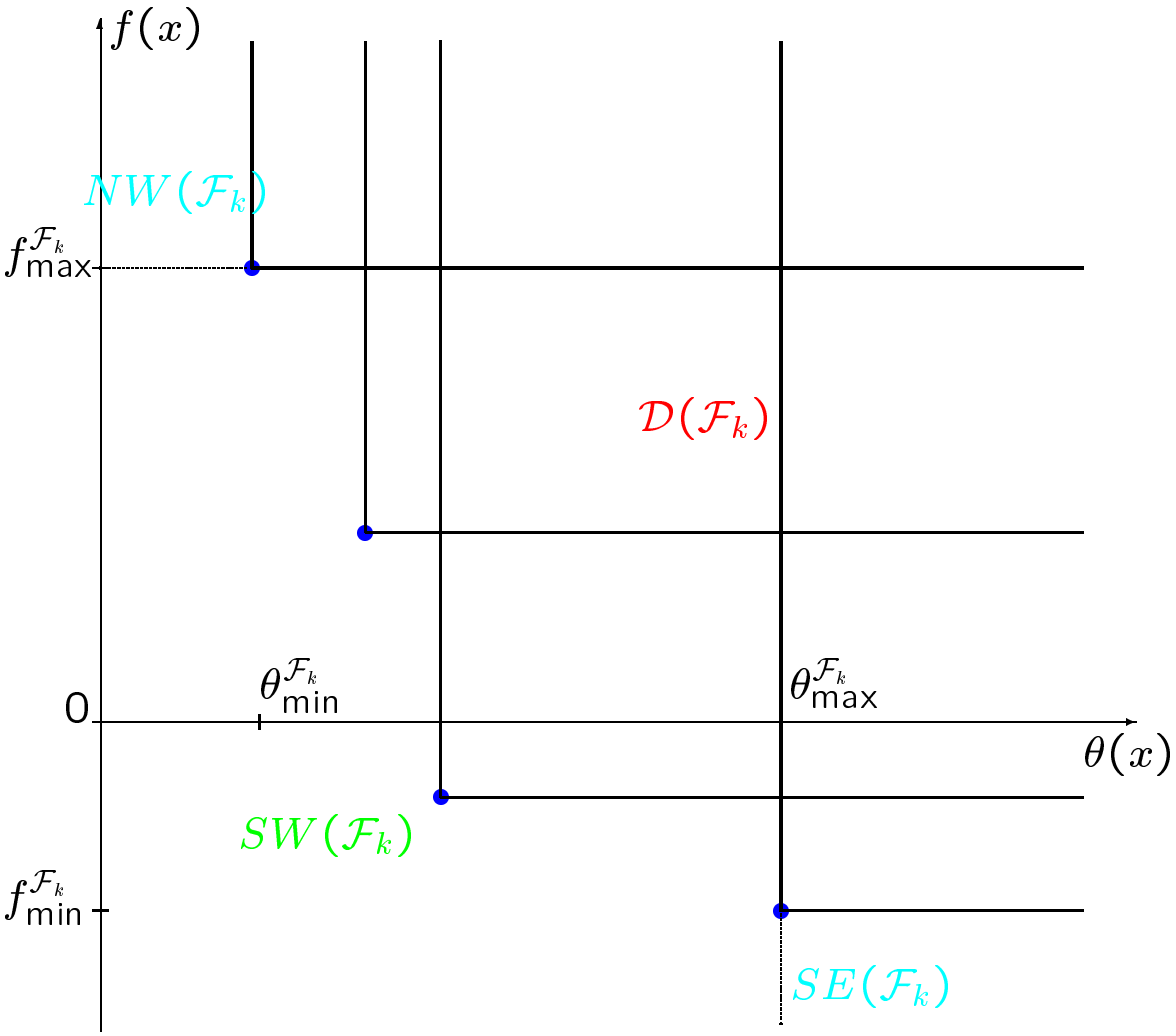


$\Rightarrow$ finitely many such points possible

(note: margin)

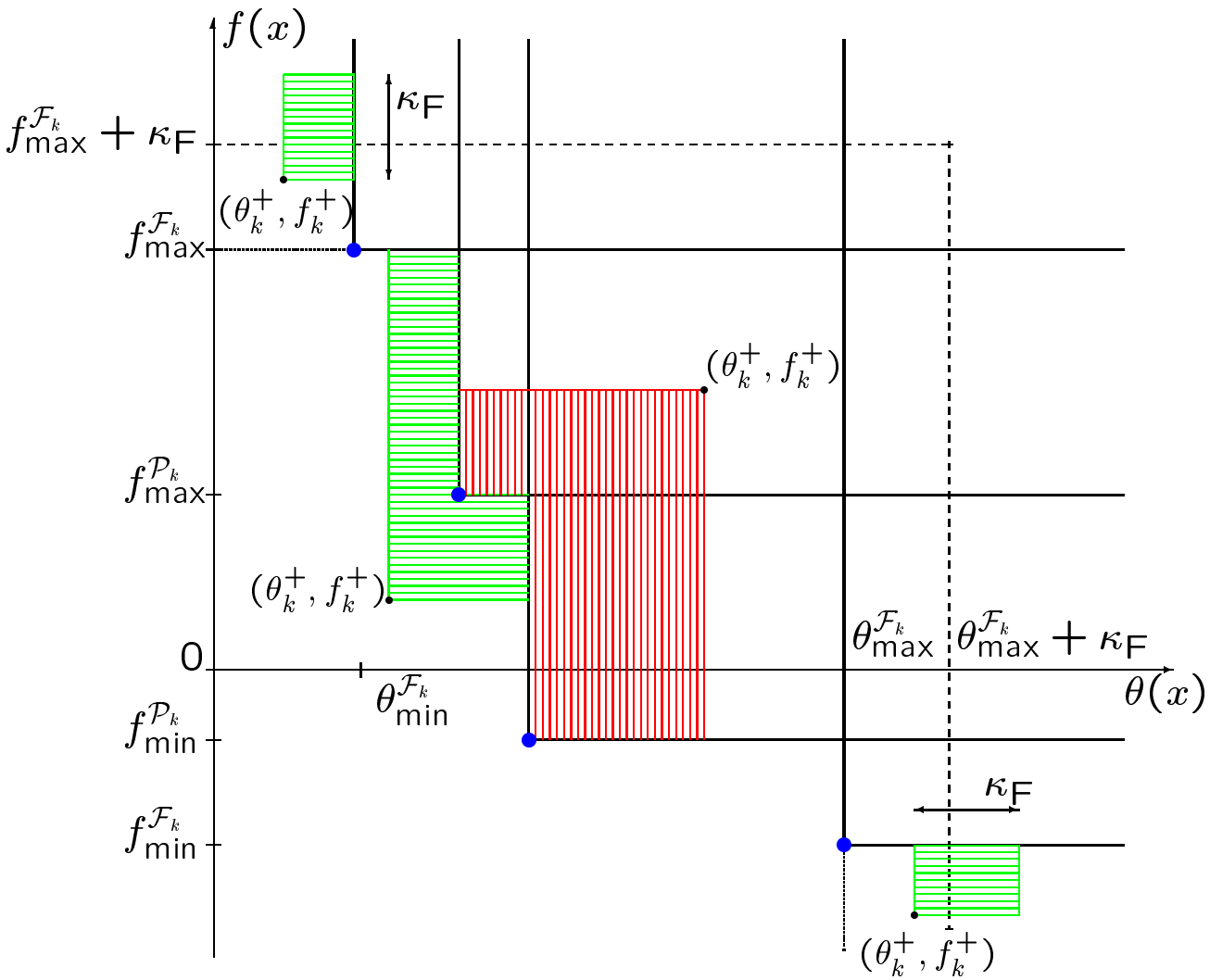
Another idea. . .

Define 4 areas in the (θ, f) space:



Area contributed to the filter

Measure α_k, the area contributed by trial point x_k^+ to the forbidden zone of the filter:



Accepting the trial point

A monotone acceptance rule:

$$\alpha_k \geq \gamma_{\mathcal{F}}(\theta_k^+)^2$$

A non-monotone acceptance rule:

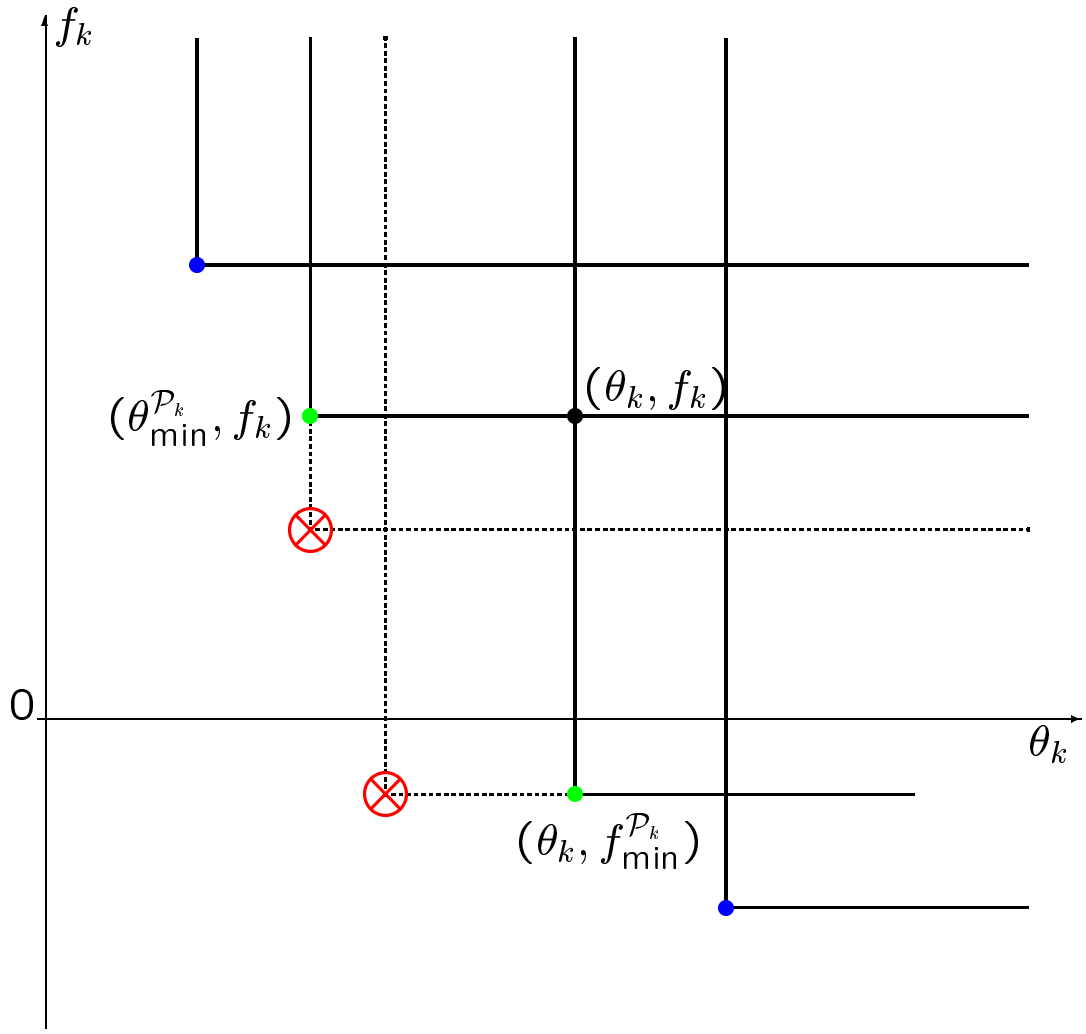
$$\sum_{\substack{j=r(k)+1 \\ j\in\mathcal{U}}}^{k} \alpha_{p(j)} + \alpha_k \geq \gamma_{\mathcal{F}} \left[\sum_{\substack{j=r(k)+1 \\ j\in\mathcal{U}}}^{k} \theta_j^2 + (\theta_k^+)^2 \right]$$

$r(k)$: a reference iteration in the past

$\mathcal{U}$: the iterations where the filter is updated

Updating the filter

Remove dominated points. But . . .



So what?

Global convergence:

There exists a first-order critical limit point of $\{x_k\}$.

Features :

- non-monotone $\Rightarrow$ avoid Maratos effect
  (in progress. . .)

- no margin
  (alternative theory for the monotone case)

Thank you for your attention