New developements in nonlinear programming with perspectives for management sciences

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Outline

1. Filter methods
   - Constrained optimization
   - Feasibility and least-squares problems
   - Unconstrained and bound-constrained optimization
2. Multilevel algorithms
3. Regularization techniques
4. Non-parametric estimation
5. Conclusions

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The problem

The general nonlinear programming problem:

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & c_{\mathcal{E}}(x) = 0 \\ & c_{\mathcal{I}}(x) \geq 0, \end{array}$$

for $x \in \mathbb{R}^n$, f and c smooth.

Solution algorithms are iterative ($\{x_k\}$) and based on Newton's method (or variant)

very many applications (also in CMS)

Issues: reliability, availability, efficiency

Our objective

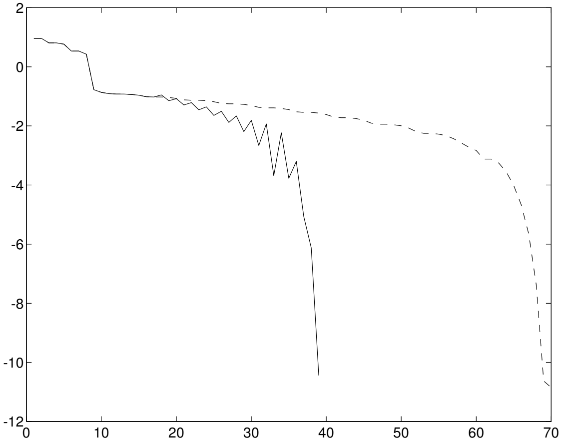
In this talk,

A glimpse into recent (and exciting) new developments

Filter methods

Filter methods

Non-monotonicity



Non-monotonicity often pays!

Introducing the filter

Constrained optimization :

use a good method (SQP) to compute a step s_k from x_k

Ideally

- reduce the objective function $f(x)$
- reduce the constraint violation $\theta(x)$

☺ two potentially conflicting aims ☹

▶ Filter method

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▶ Filter method

Accepting a new iterate

Idea of Fletcher and Leyffer

Replace the question

What is a better point ?

by

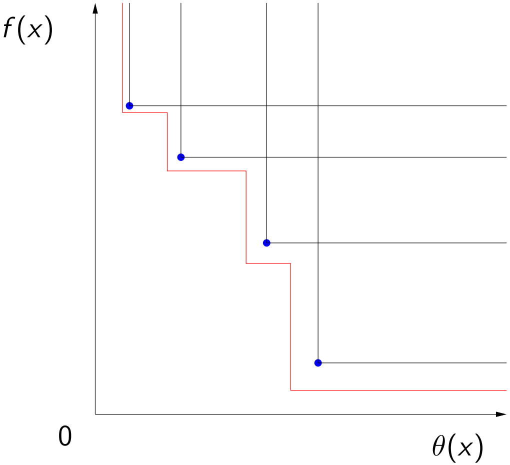
What is a worse point ?

Of course, y is "worse" than x if it is dominated by x, i.e., when

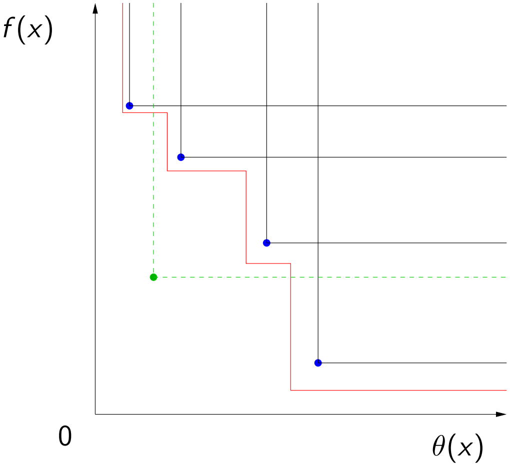
$$f(x) \leq f(y) \quad \text{and} \quad \theta(x) \leq \theta(y)$$

Accept or reject a trial point ?

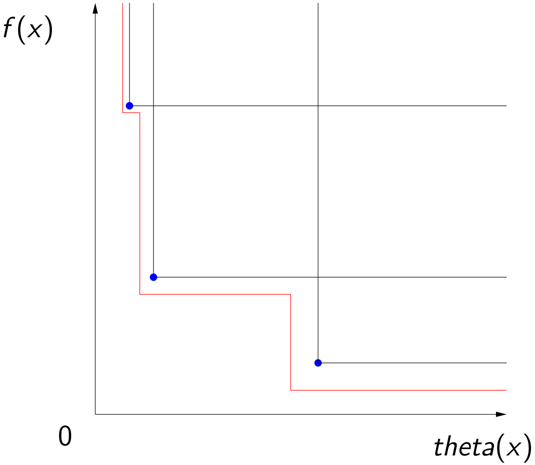
View of a standard filter



View of a standard filter



View of a standard filter



Handling many objectives

Gould, Leyffer and T.

Feasibility

Find x such that

$$c_{\mathcal{E}}(x) = 0 \quad \text{and} \quad c_{\mathcal{I}}(x) \geq 0$$

Least-squares

Find x such that

$$\min_{x \in \mathbf{R}^n} \sum_{i \in \mathcal{E} \cup \mathcal{I}} \theta_i(x)^2$$

▶ More dimensions in the filter space . . .

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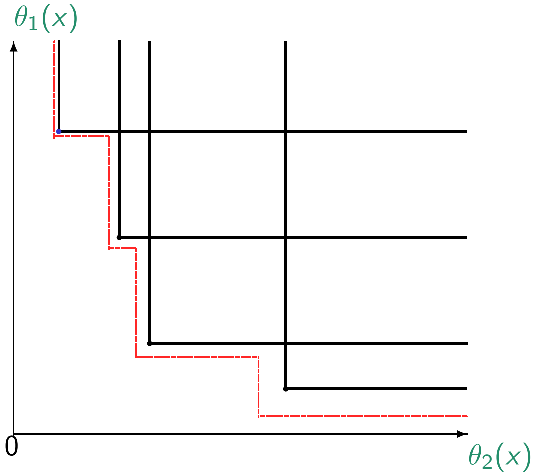
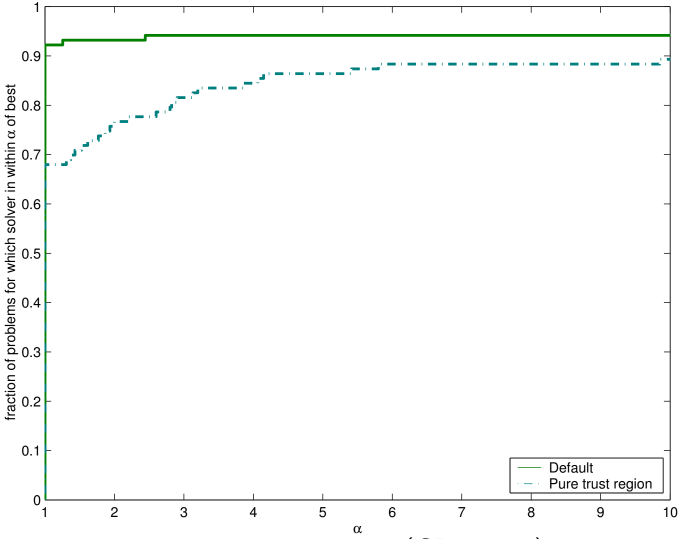
The obvious picture



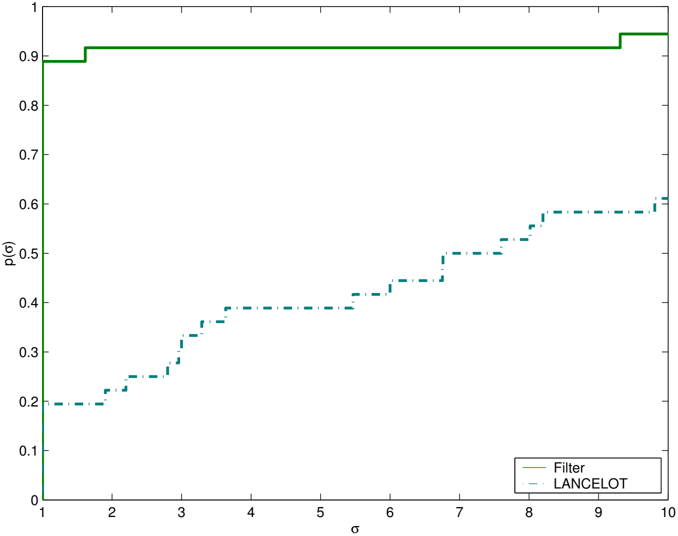
(full dimension vs. grouping)

Numerical experience (1)



Filter vs. trust-region (CPU time)

Numerical experience (2)



Filter vs. LANCELOT B (CPU time)

The other extreme case

Gould, Sainvitu and T.

Unconstrained optimization

$$\min_{x \in \mathbf{R}^n} f(x)$$

Simple-bound constrained optimization

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & l \leq x \leq u \end{array}$$

Combined methods:

Trust-region + filter + projected gradient

▶ Multidimensional filter

A gradient multidimensional filter

Accept x_k^+ more often

A point x dominates a point y whenever

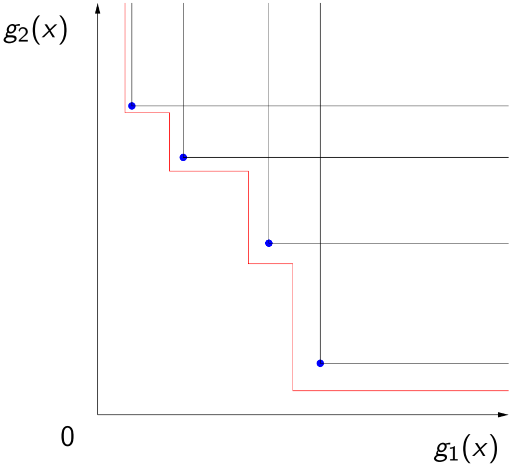
$$|g_i(x)| \leq |g_i(y)| \quad \forall\, i = 1, \ldots, n$$

Remember non-dominated points $\Rightarrow$ FILTER

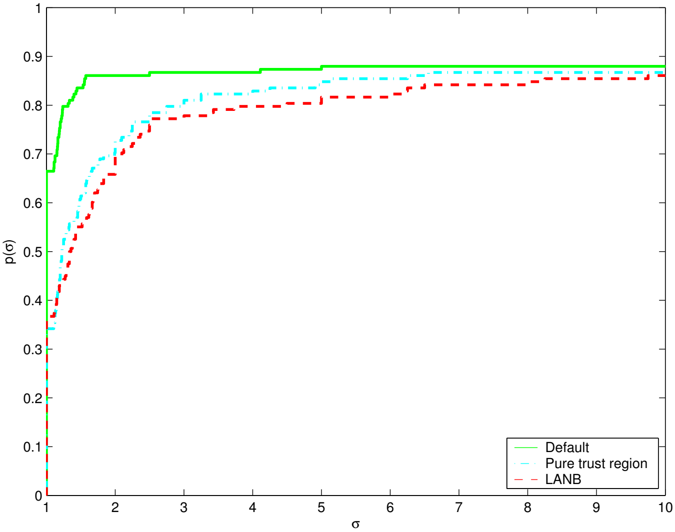
Accept a new iterate ?

if it is not dominated by any other iterate in the filter

Haven't we seen this before?

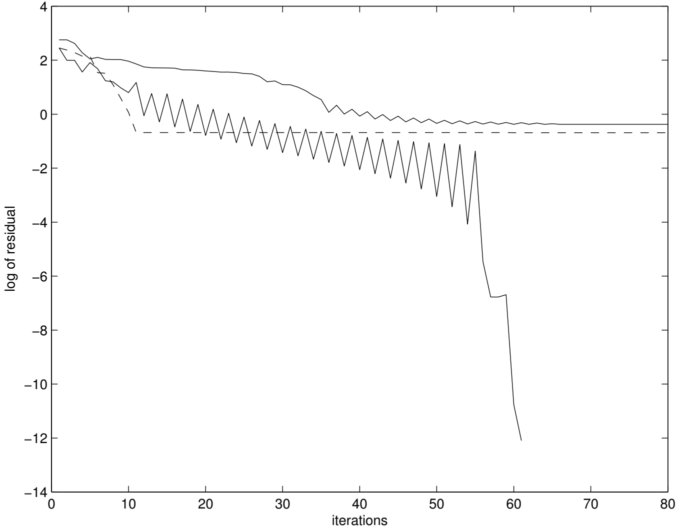


Numerical experience (unconstrained)



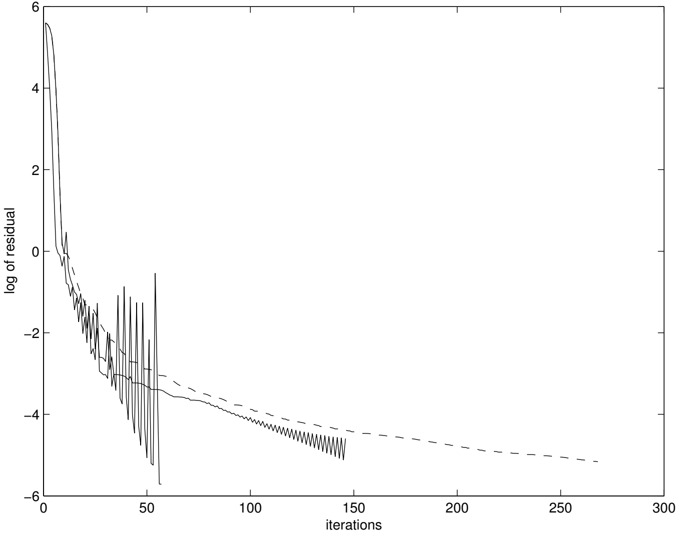
Filter vs. trust-region and LANCELOT B (iterations)

Numerical experience: HEART6



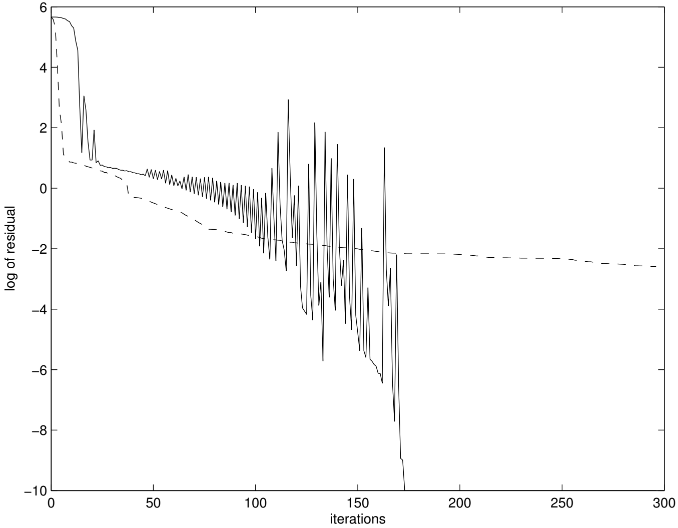
Filter vs. trust-region and LANCELOT B

Numerical experience: EXTROSNB



Filter vs. trust-region and LANCELOT B

Numerical experience: LOBSTERZ



Filter vs. trust-region

Multilevel algorithms

Multilevel algorithms

Motivation

- Optimization of continuous problems occurs in a many applications: shape optimization, data assimilation, control problems, . . .
- Recent optimization methods have been designed to cope with these problems, including multilevel/multigrid algorithms.
- These algorithms involve the computation of a hierarchy of problem descriptions, linked by known operators.

Here: focus on unconstrained/ bound-constrained optimization:

$$\min_{(x \geq 0)} f(x)$$

Hierarchy of problem descriptions

Can we use a structure of the form:

Finest problem description

Restriction $\downarrow$ R — P $\uparrow$ Prolongation

Fine problem description

Restriction $\downarrow$ R — P $\uparrow$ Prolongation

$\cdots$

Restriction $\downarrow$ R — P $\uparrow$ Prolongation

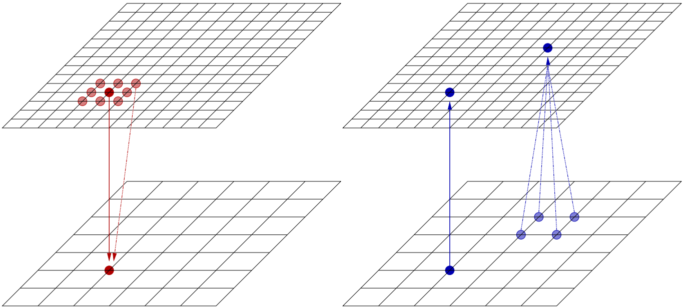
Coarse problem description

Restriction $\downarrow$ R — P $\uparrow$ Prolongation

Coarsest problem description

Grid transfer operators

$$R_i : \quad \mathbb{R}^{n_i} \quad \longrightarrow \quad \mathbb{R}^{n_{i-1}} \qquad \text{Restriction}$$
$$P_i : \quad \mathbb{R}^{n_{i-1}} \quad \longrightarrow \quad \mathbb{R}^{n_i} \qquad \text{Prolongation}$$

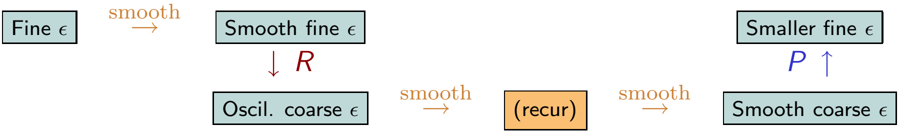


Three keys to multigrid algorithms

- oscillatory components of the error are representable on fine grids, but not on coarse grids
- iterative methods reduce oscillatory components much faster than smooth ones
- smooth on fine grids $\longrightarrow$ oscillatory on coarse ones

How to exploit these keys

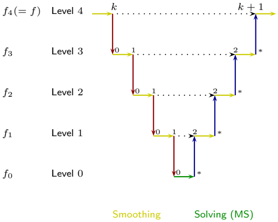
Annihilate oscillatory error level by level:

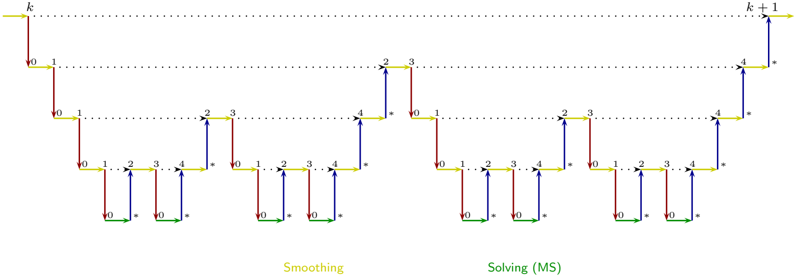


Note: P and R are not othogonal projectors!

A very efficient method for some linear systems
(when A(smooth modes) $\in$ smooth modes)

Iteration structure across levels

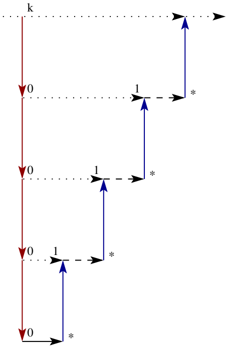




Mesh refinement, as different from. . .

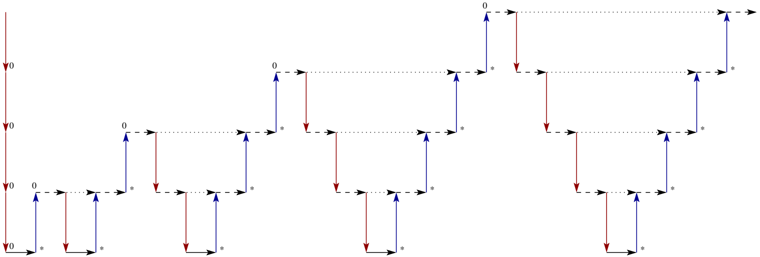
Computing good starting points:

- Solve the problem on the coarsest level
  $\Rightarrow$ Good starting point for the next fine level
- Do the same on each level
  $\Rightarrow$ Good starting point for the finest level
- Finally solve the problem on the finest level



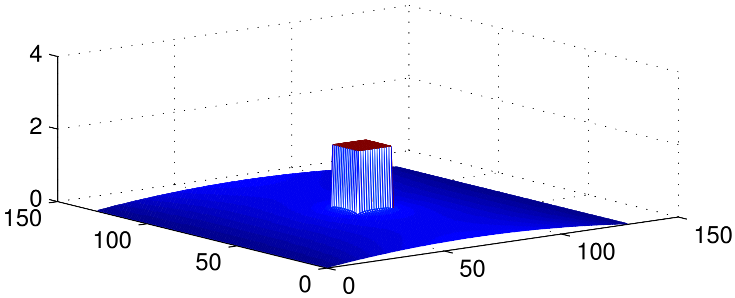
. . . V-cycles and Full Multigrid (FMG)

- FMG : Combination of mesh refinement and V-cycles

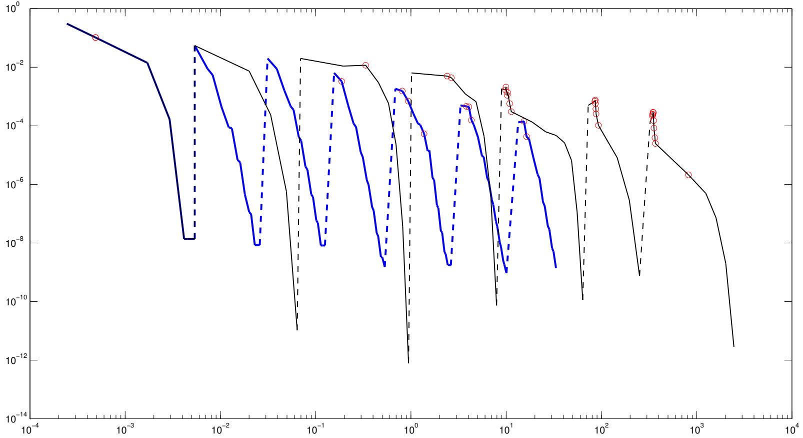


Results on an obstacle problem ($n = 127^2$, 6 levels)

	unconstrained			bound-constrained	
	Mesh ref.	$RMTR_2$	$RMTR_\infty$	Mesh ref.	$RMTR_\infty$
nit	1057	23	10	2768	214
nf	23	38	15	649	240
ng	16	28	14	640	236
nH	17	20	6	32	101



Equivalent iterations on the minimum surface problem



mesh-refinement FMG

Regularization techniques

Regularization techniques

Unconstrained optimization — a "mature" area?

$$\underset{x \in \mathbf{R}^n}{\text{minimize}} \quad f(x) \quad \text{where} \quad f \in C^1 \quad (\text{maybe} \quad C^2 \;)$$

Currently two main competing (but similar) methodologies

- **Linesearch methods**
  - compute a descent direction s_k from x_k
  - set $x_{k+1} = x_k + \alpha_k s_k$ to improve f
- **Trust-region methods**
  - compute a step s_k from x_k to improve a model m_k of f within the trust-region $\|s\| \leq \Delta$
  - set $x_{k+1} = x_k + s_k$ if m_k and f "agree" at $x_k + s_k$
  - otherwise set $x_{k+1} = x_k$ and reduce the radius Δ

Is there anything more to say?

Recently, Nesterov and Polyak (2006) observed the following: if

- f has gradient g and globally Lipschitz continuous Hessian H with constant $2L$

Taylor, Cauchy-Schwarz and Lipschitz imply

$$\begin{aligned} f(x+s) &= f(x) + \langle s, g(x)\rangle + \tfrac{1}{2}\langle s, H(x)s\rangle \\ &\quad + \int_0^1 (1-\alpha)\langle s, [H(x+\alpha s) - H(x)]s\rangle \, d\alpha \\ &\leq \underbrace{f(x) + \langle s, g(x)\rangle + \tfrac{1}{2}\langle s, H(x)s\rangle + \tfrac{1}{3}L\|s\|_2^3}_{m(s)} \end{aligned}$$

$\Longrightarrow$ reducing m from $s=0$ improves f since $m(0)=f(x)$.

The cubic regularization

Change from

$$\min_s \quad f(x) + \langle s, g(x)\rangle + \tfrac{1}{2}\langle s, H(x)s\rangle \;\; \text{s.t.} \;\; \|s\| \le \Delta$$

to

$$\min_s \quad f(x) + \langle s, g(x)\rangle + \tfrac{1}{2}\langle s, H(x)s\rangle + \tfrac{1}{3}\sigma\|s\|^3$$

σ is the (adaptive) regularization parameter

The main features of adaptive cubic regularization

And the result is. . .

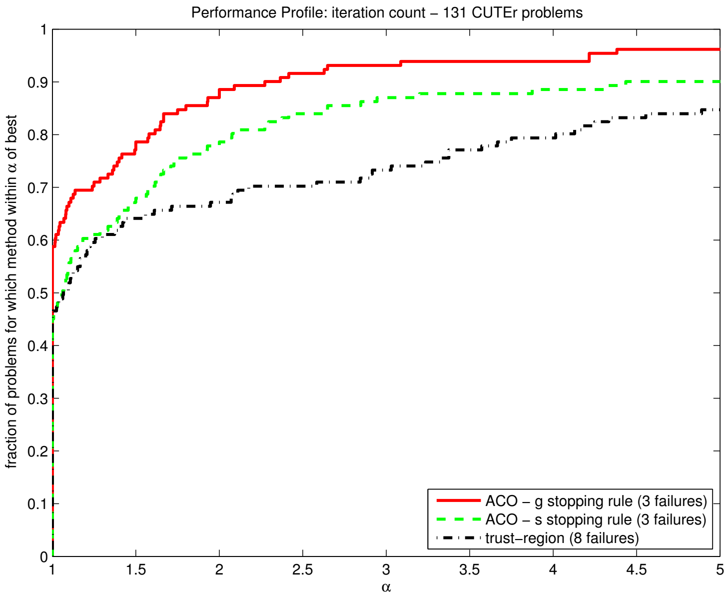
longer steps on ill-conditioned problems

similar (very satisfactory) convergence analysis

best known worst-case complexity for nonconvex problems

excellent performance and reliability

Numerical experience — small problems using Matlab



Non-parametric estimation

Non-parametric estimation

The context and problem

Agent decision modelling

Random utility models

Max likelihood parameter estimation

- Problem: Value of parameter(s) sometimes unrealistic!
  Example: negative VOT. . .
- symmetric distribution assumed in random parts of utility!

Using advanced optimization tools. . .

What can we do?

- avoid irrealistic distribution assumptions
- estimate non-parametric distributions from the data
- need for an efficient representation of non-parametric distributions

cubic spline representation of inverse CDF

$\Rightarrow$ order constraint on the knot values in likelihood maximization!

efficient projection based trust-region algorithm

And the result is. . .

Excellent estimation of asymmetric distribution

A nice application on the behaviour of the BoJ of the FX market

▶ See Fabian Bastin's talk!◀

Thus . . .

Good use of advanced optimization = progress in CMS

Conclusions

- a vibrant field with new perspectives
- expected: more reliable and more efficient methods
- ... available to a larger public
- ... and applications in more challenging domains
- CMS continues to be the source of
  - interesting applications
  - methodologicaal inspiration

Thank you for your attention !

(see `http://perso.fundp.ac.be/~phtoint/publications.html` for references)