Non-monotonicity and filter methods in nonlinear optimization

Philippe Toint

(with N. Gould, C. Sainvitu and S. Leyffer)

philippe.toint@fundp.ac.be

Department of Mathematics, University of Namur, Namur

Nonlinear optimization

The general nonlinear programming problem:

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & c_{\mathcal{E}}(x) = 0 \\ & c_{\mathcal{I}}(x) \geq 0, \end{array}$$

for $x \in \mathbb{R}^n$, f and c smooth.

Solution algorithms are

- iterative ($\{x_k\}$)
- based on Newton's method (or variant)

$\Rightarrow$ global convergence issues

Monotonicity (1)

Global convergence theoretically ensured by

- some global measure . . .
  - unconstrained : $f(x_k)$
  - constrained : merit function at x_k
- . . . with strong monotonic behaviour

(Lyapunov function)

Also practically enforced by

- algorithmic safeguards around Newton method
  (linesearches, trust regions)

Plan
→ *Monotonicity*
● Constrained opt.
● Unconstrained opt.

Monotonicity (2)

But

classical safeguards limit efficiency!

Question :

design less obstructive safeguards

while

- ensuring better numerical performance (the Newton Liberation Front !)
- continuing to guarantee global convergence properties

Non-monotone methods

Typically:

- abandon strict monotonicity of usual measures
- but insist on average behaviour

linesearch:

- Chamberlain, Powell, Lemarechal, Pedersen (1982)
- Grippo, Lampariello, Lucidi, Facchinei (1986, 1989, 1991, 1992,. . .)
- Panier, Tits, Bonnans, Zhou (1991, 1992), T. (1996), . . .

trust region:

- Deng, Xiao, Zhou (1992, 1993, 1994, 1995)
- T. (1994, 1997), Conn, Gould, T. (2000)
- Ke, Han, Liu (1995, 1996), Burke, Weigmann (1997), Yuan (1999), . . .

Non-monotone trust-regions

Idea:

$$\boxed{f(x_{k+1}) < f(x_k) \text{ replaced by } f(x_{k+1}) < f_{r(k)}}$$

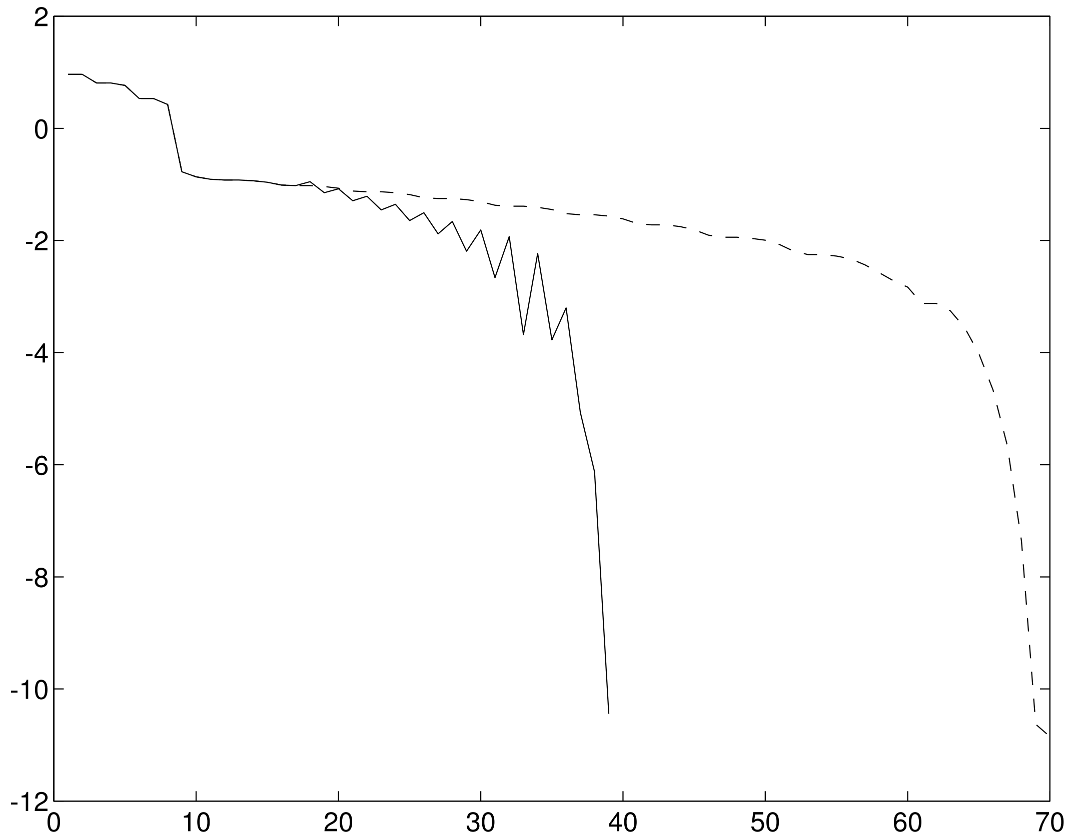
with

$$\boxed{f_{r(k)} < f_{r(k-1)}}$$

Further issues:

- suitably define $r(k)$
- adapt the trust-region algorithm: also compare achieved and predicted reductions since reference iteration

An unconstrained example



Monotone and non-monotone TR

A code: LANCELOT B

Plan

→ ***Monotonicity***

- Constrained opt.
- Unconstrained opt.

Introducing the filter

A fruitful alternative: $\boxed{\text{filter methods}}$

Constrained optimization :

using the SQP step, at the same time:

- reduce the objective function $f(x)$
- reduce constraint violation $\theta(x)$

$\Rightarrow$ CONFLICT

The filter point of view

Fletcher and Leyffer replace question:

What is a better point?

by:

What is a worse point?

Of course, y is worse than x when

$$f(x) \leq f(y) \;\text{ and }\; \theta(x) \leq \theta(y)$$

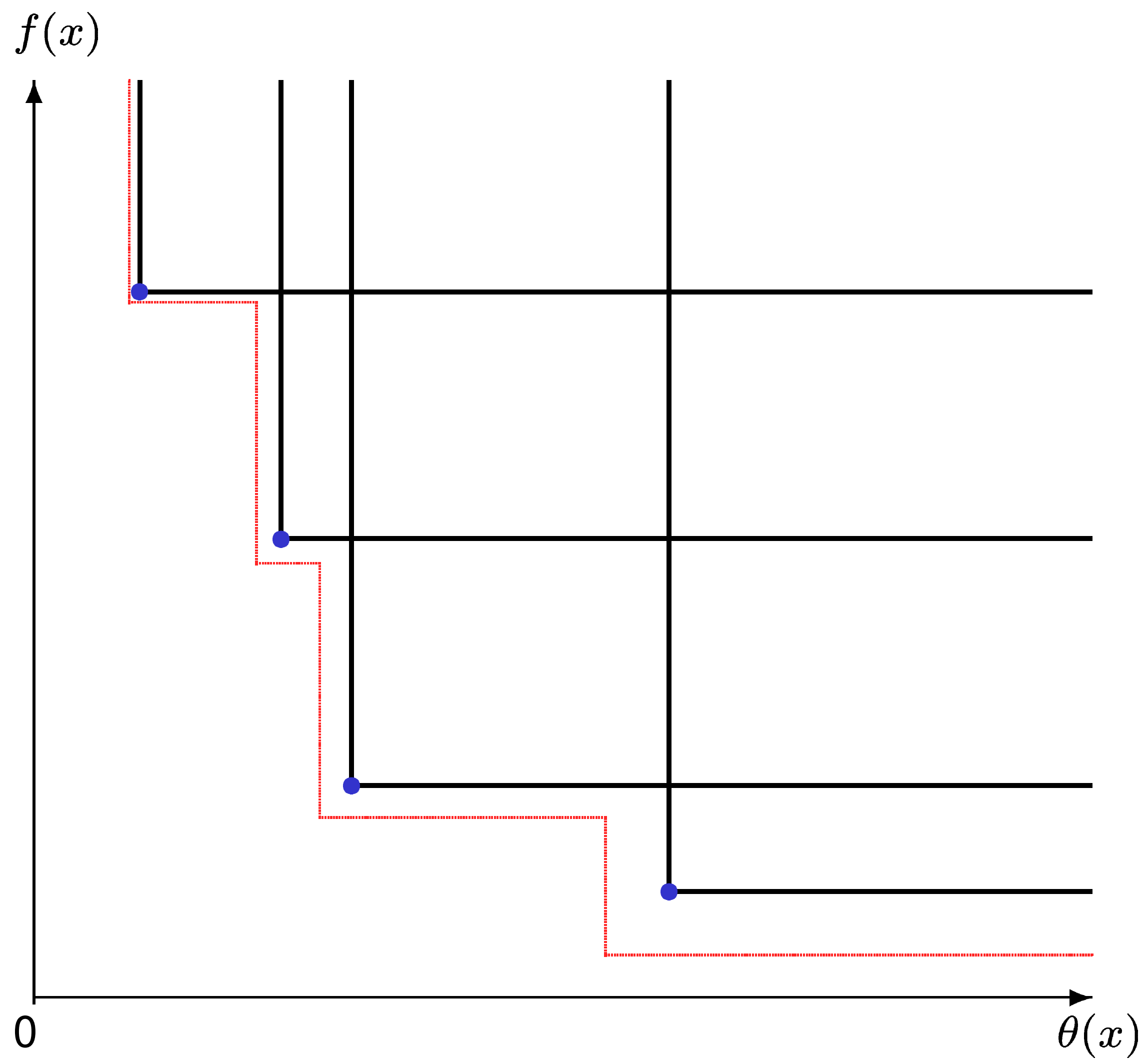
(y is dominated by x)

When is $x_k + s_k$ acceptable?

The standard filter

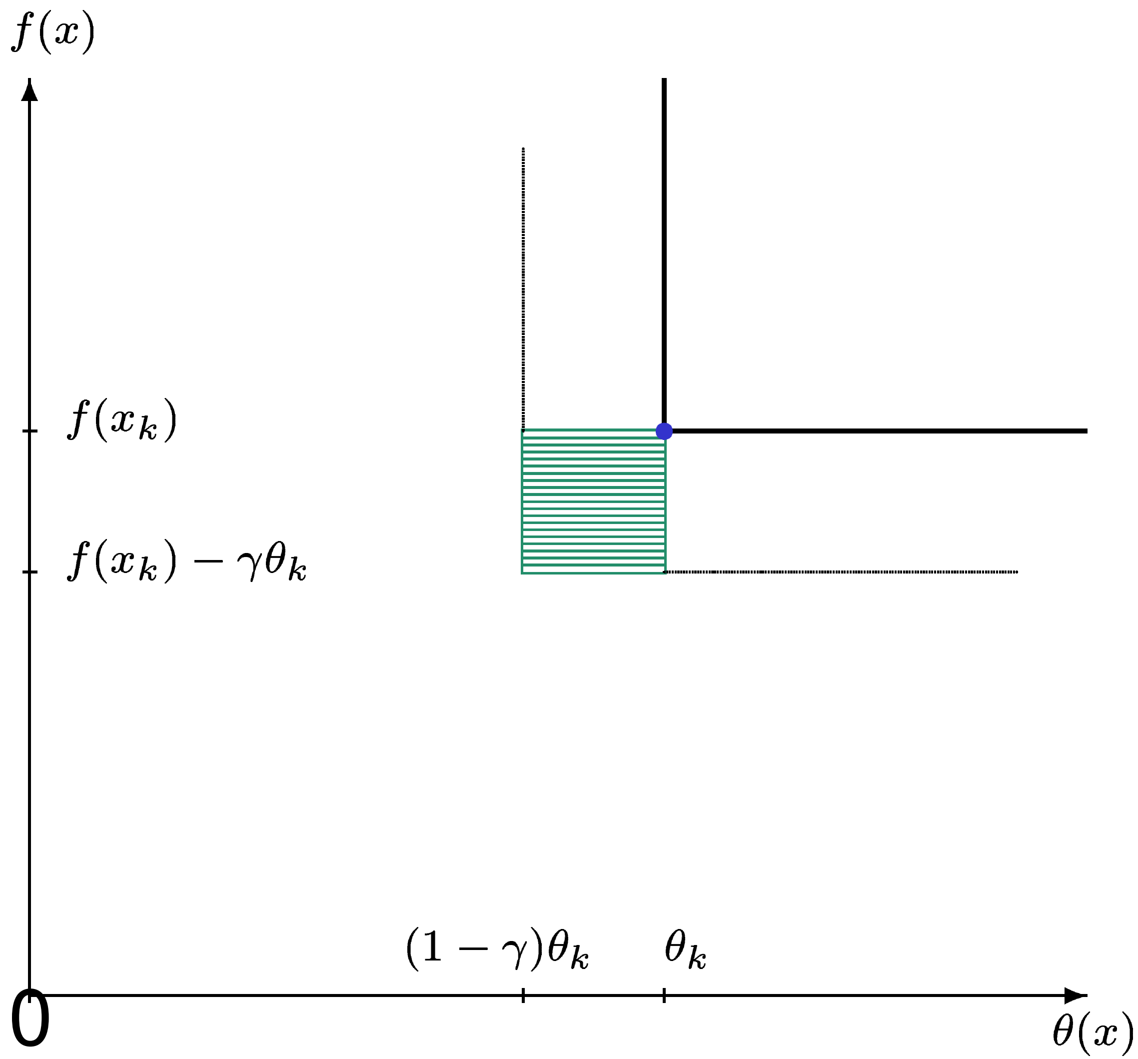
Idea: accept non-dominated points

no monotonicity of merit function implied



Filling up the standard filter

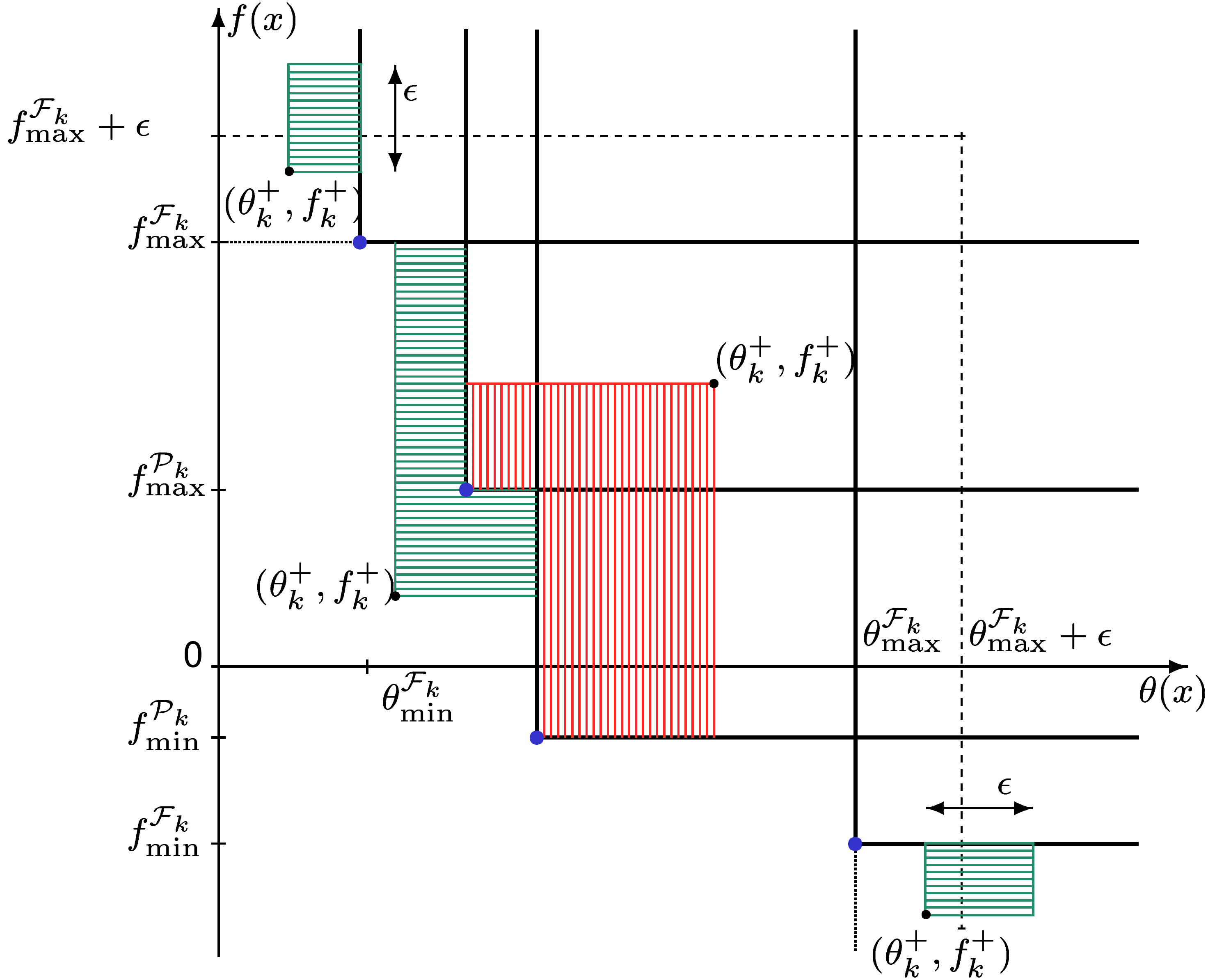
Note: filter area is bounded in the (f, θ) space!



$\Rightarrow$ filter area monotonically decreasing

Another idea...

Measure α_k, the area contributed by x_k^+



Plan
- Monotonicity
- → ***Constrained opt.***
- Unconstrained opt.

The University of Namur
www.fundp.ac.be

Accepting the trial point?

An (areawise) monotone acceptance rule:

$$\alpha_k \geq \gamma_{\mathcal{F}} (\theta_k^+)^2$$

A non-monotone acceptance rule:

$$\sum_{j=r(k)+1, j \in \mathcal{U}}^{k} \alpha_{p(j)} + \alpha_k \geq \gamma_{\mathcal{F}} \left[\sum_{j=r(k)+1, j \in \mathcal{U}}^{k} \theta_j^2 + (\theta_k^+)^2 \right]$$

$r(k)$: reference iteration, $p(j)$: predecessor of j

$\mathcal{U}$: the iterations where the filter is updated

sufficient area reduction since reference iteration

The (unconst.) feasibility problem

Feasibility

Find x such that

$$c(x) \geq 0$$

$$e(x) = 0$$

for general smooth c and e.

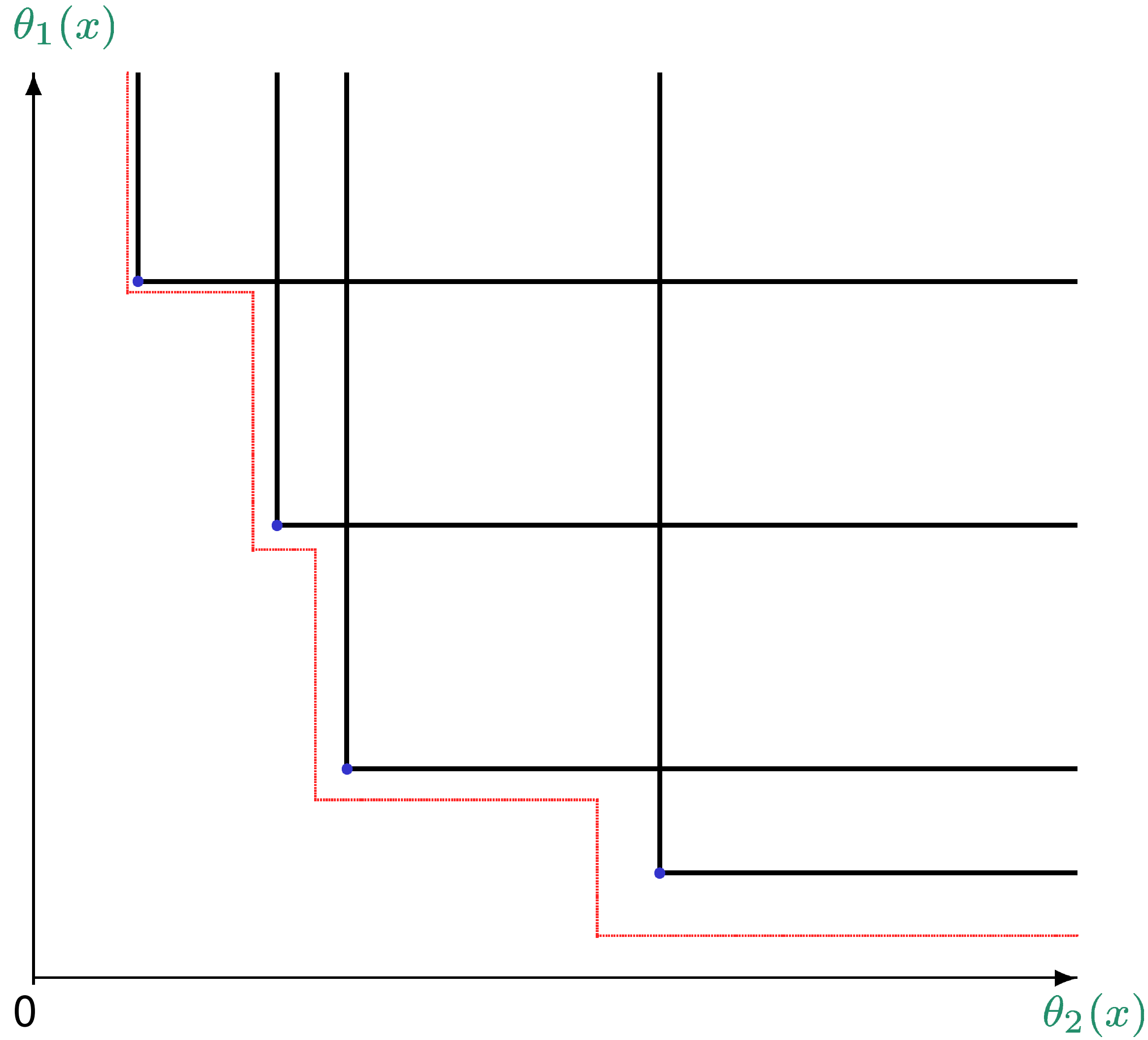
Least-squares

Find x such that

$$\min \sum \theta_i^2$$

A multidimensional filter (1)

(Simple) idea: more dimensions in filter space



(full dimension vs. grouping)

A multidimensional filter (2)

Additionally

- possibly consider unsigned filter entries
- use TR algorithm when
  - trial point unacceptable
  - convergence to non-zero solution

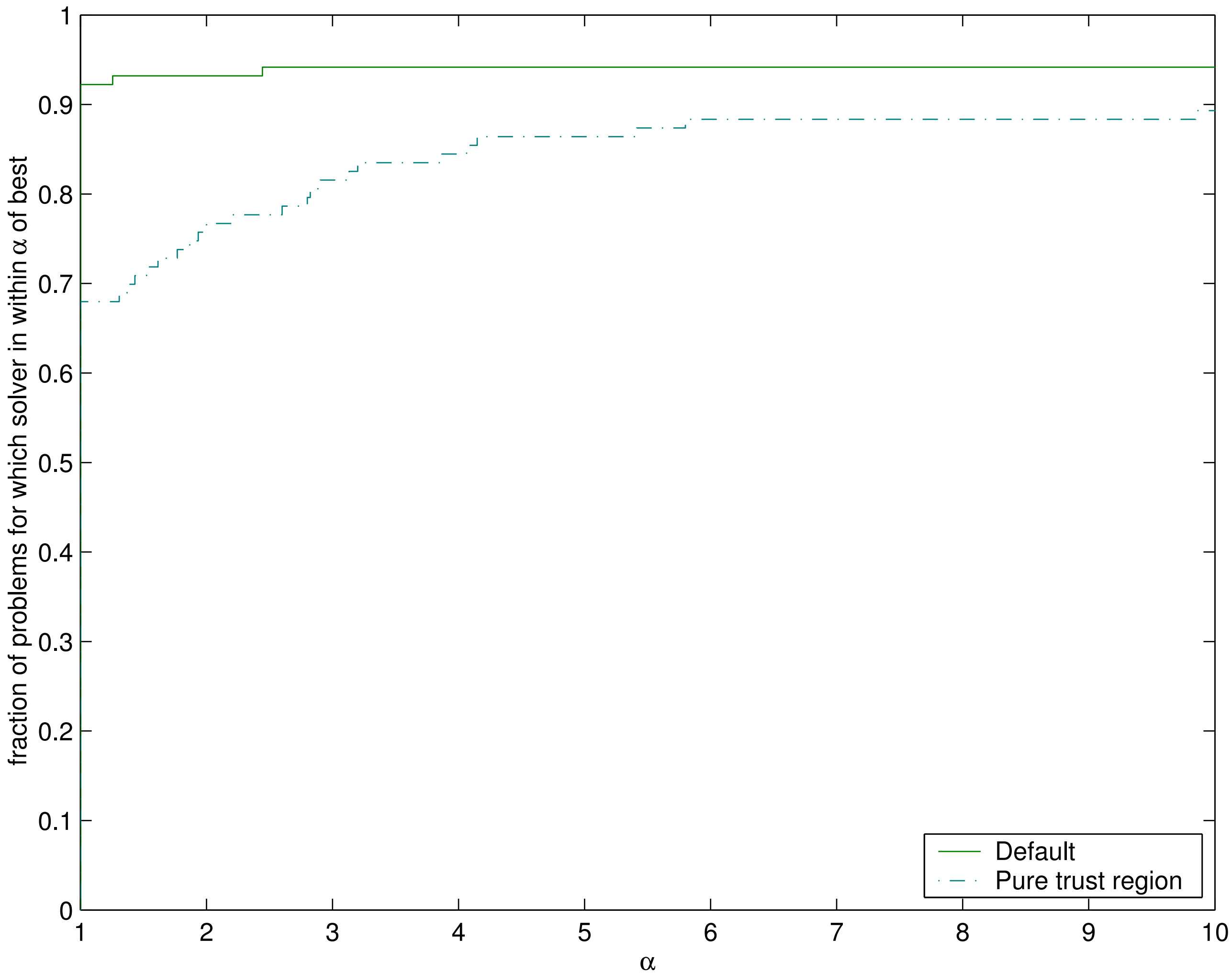
($\Rightarrow$ “internal” restoration)

sound convergence theory

Numerical experience: FILTRANE

- Fortran 95 package
- large scale problems (CUTEr interface)
- includes several variants of the method
  - signed/unsigned filters
  - Gauss-Newton, Newton or adaptive models
  - pure trust-region option
  - uses preconditioned conjugate-gradients + Lanczos for subproblem solution
- part of the GALAHAD library

Numerical experience (1)



Filter vs. trust-region (CPU time)

Plan

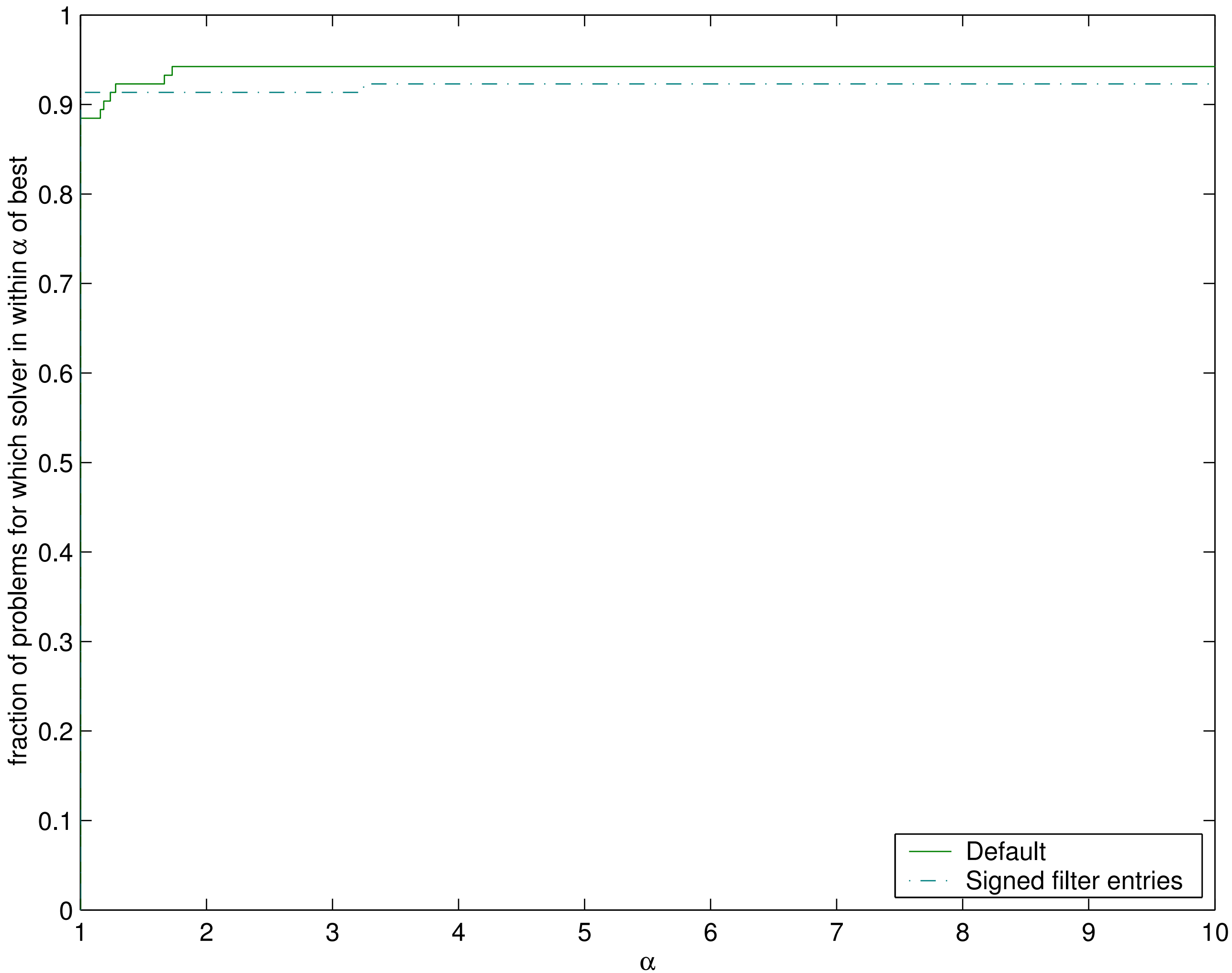
- Monotonicity
- Constrained opt.

→ ***Unconstrained opt.***

The University of Namur

www.fundp.ac.be

Numerical experience (2)



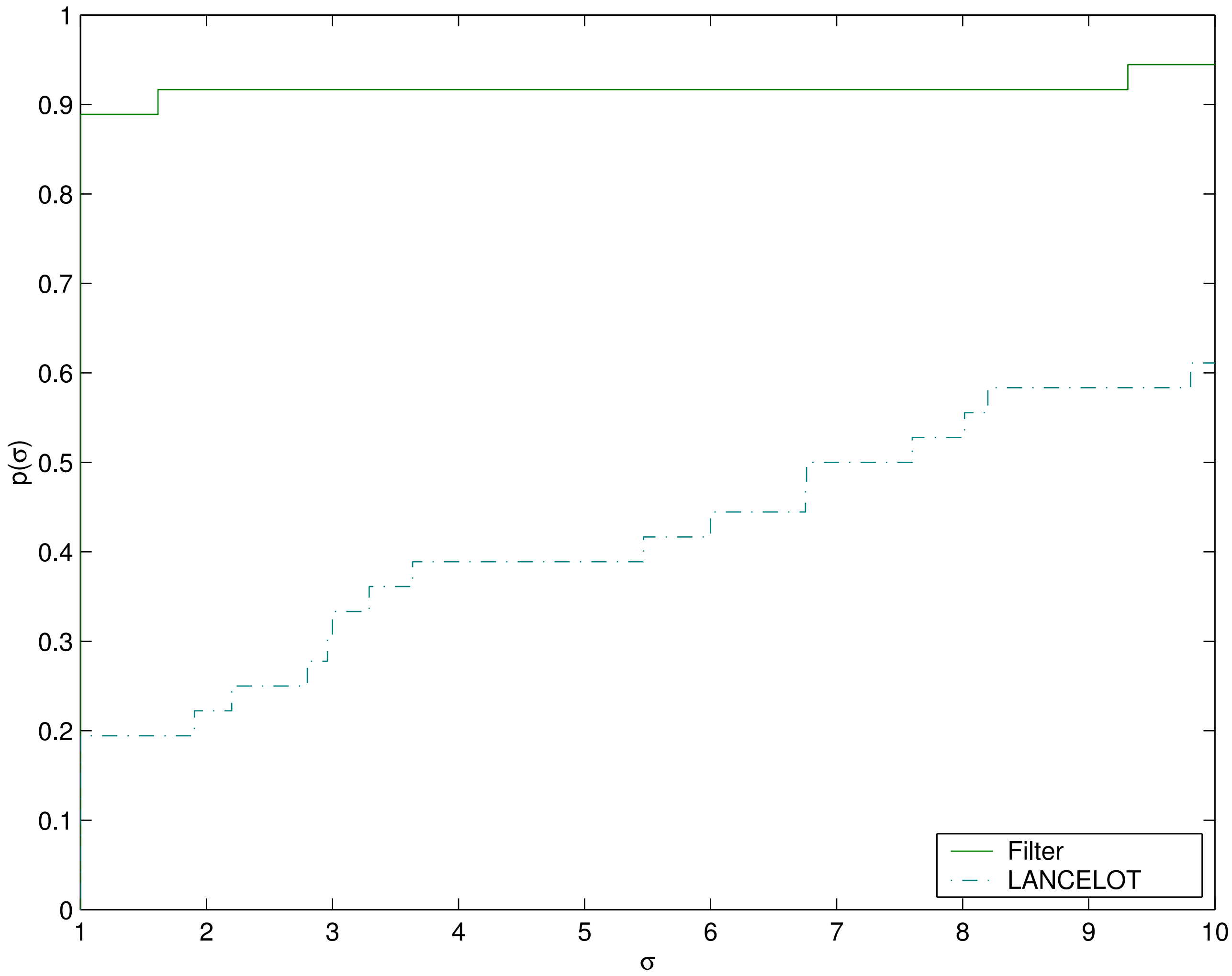
Allowing unsigned filter entries (CPU time)

Plan

- Monotonicity
- Constrained opt.

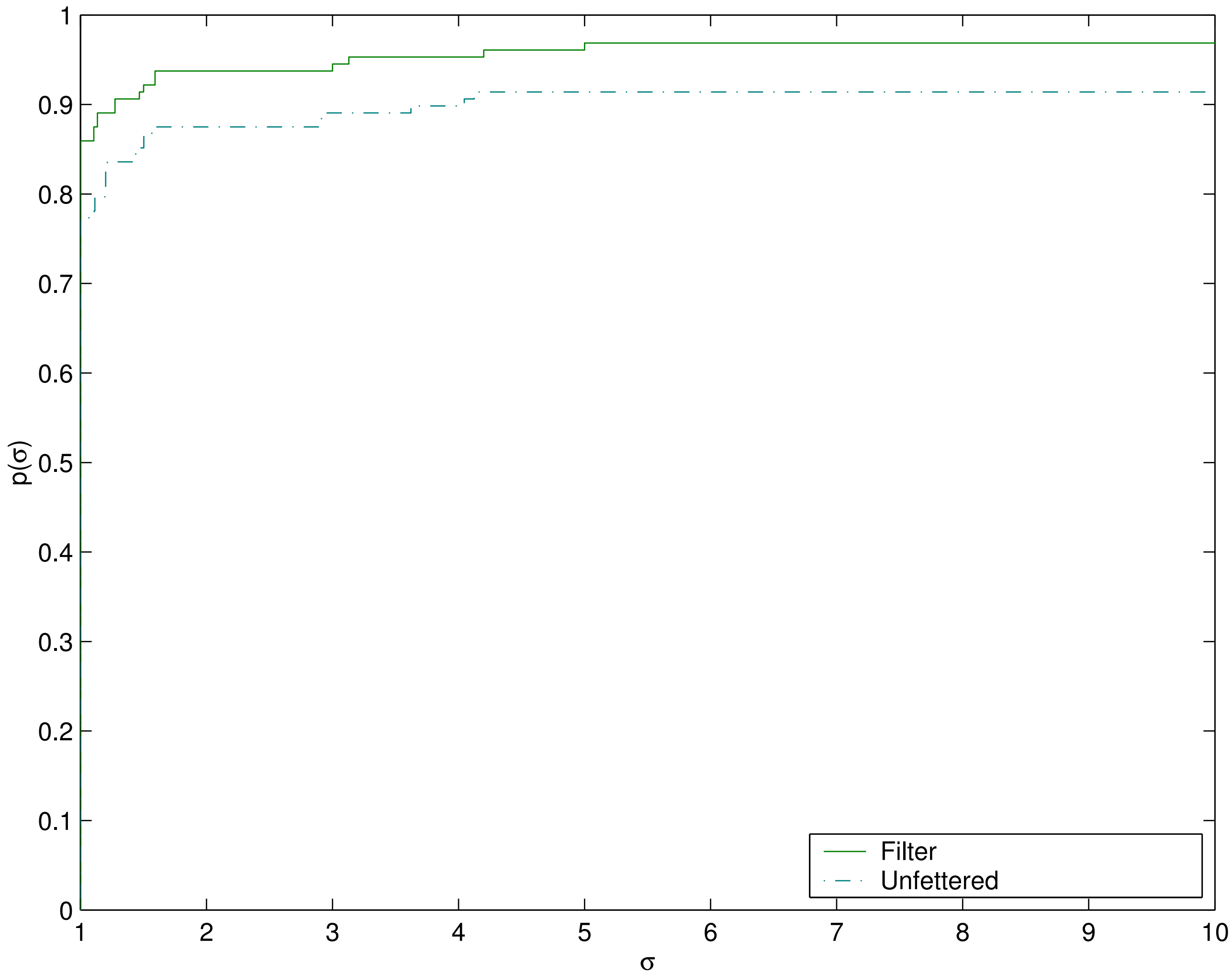
→ ***Unconstrained opt.***

Numerical experience (3)



Filter vs. LANCELOT B (CPU time)

Numerical experience (4)



Filter vs. free Newton (CPU time)

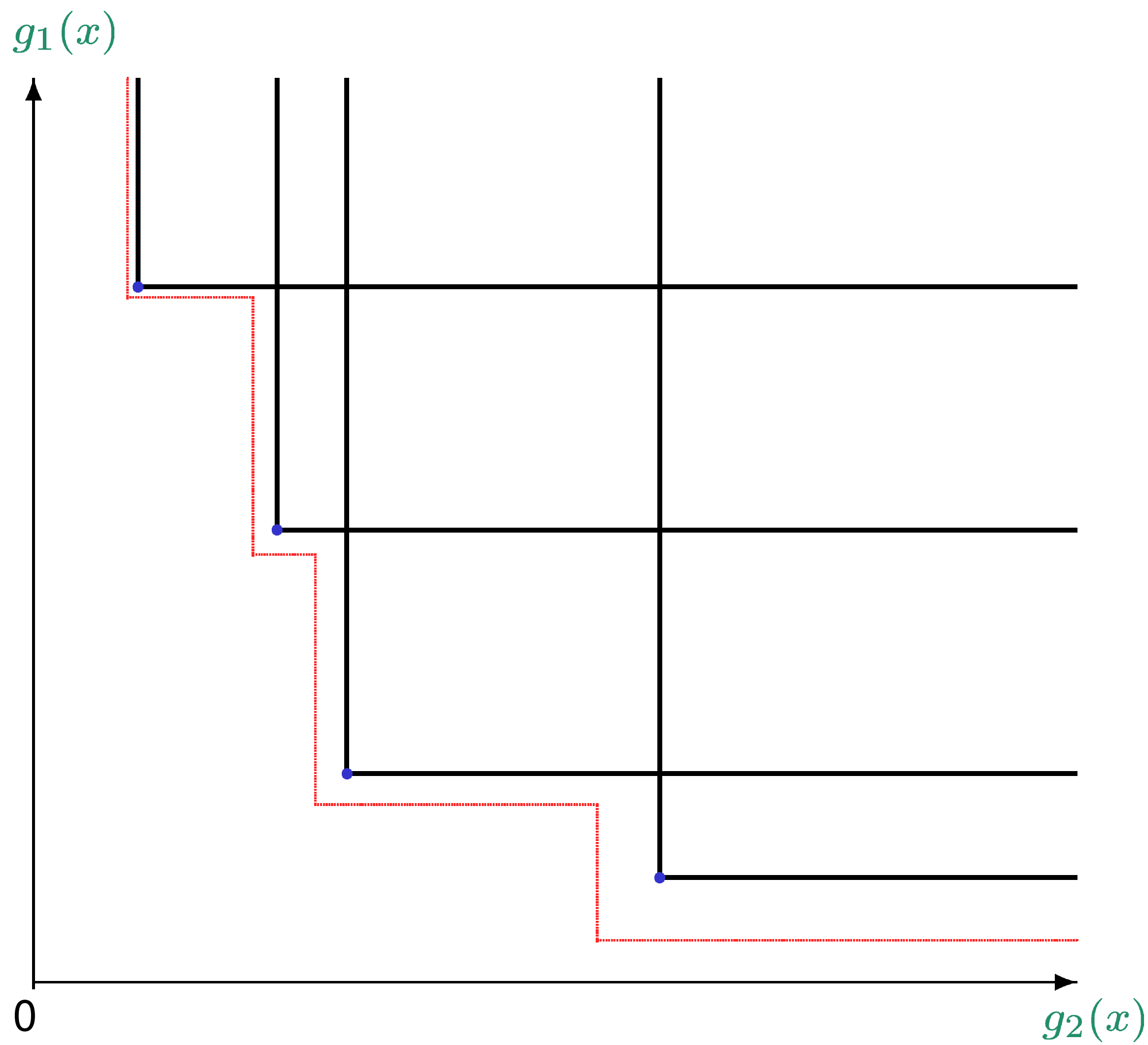
Plan

- Monotonicity
- Constrained opt.

→ ***Unconstrained opt.***

Filter for unconstrained opt.

Again simple idea: use g_i instead of θ_i



(full dimension vs. grouping)

A few complications…

But …

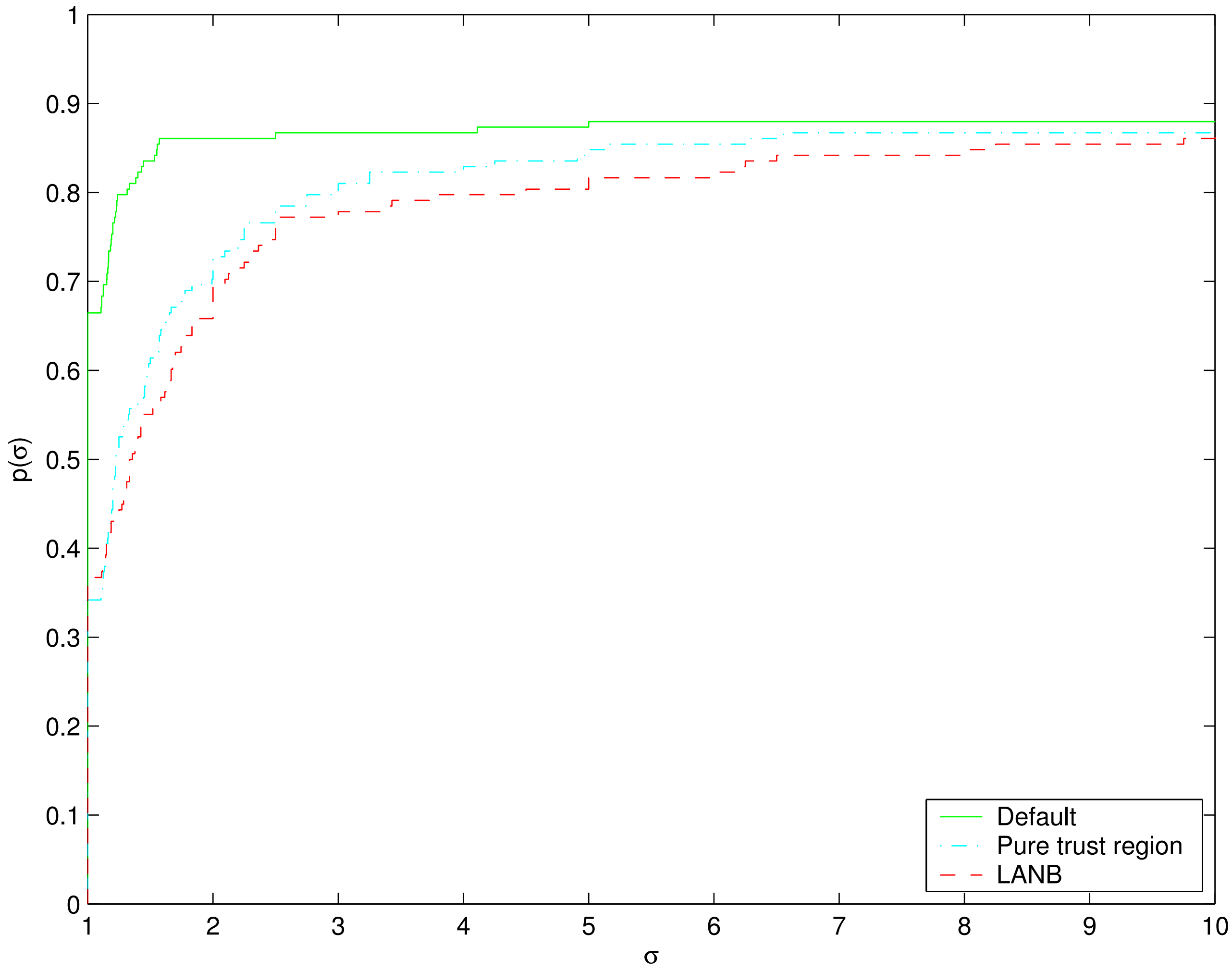
$g(x) = 0$ not sufficient for nonconvex problems!

When negative curvature found:

- reset filter
- set upper bound on acceptable $f(x)$

reasonable convergence theory

Numerical experience (1)

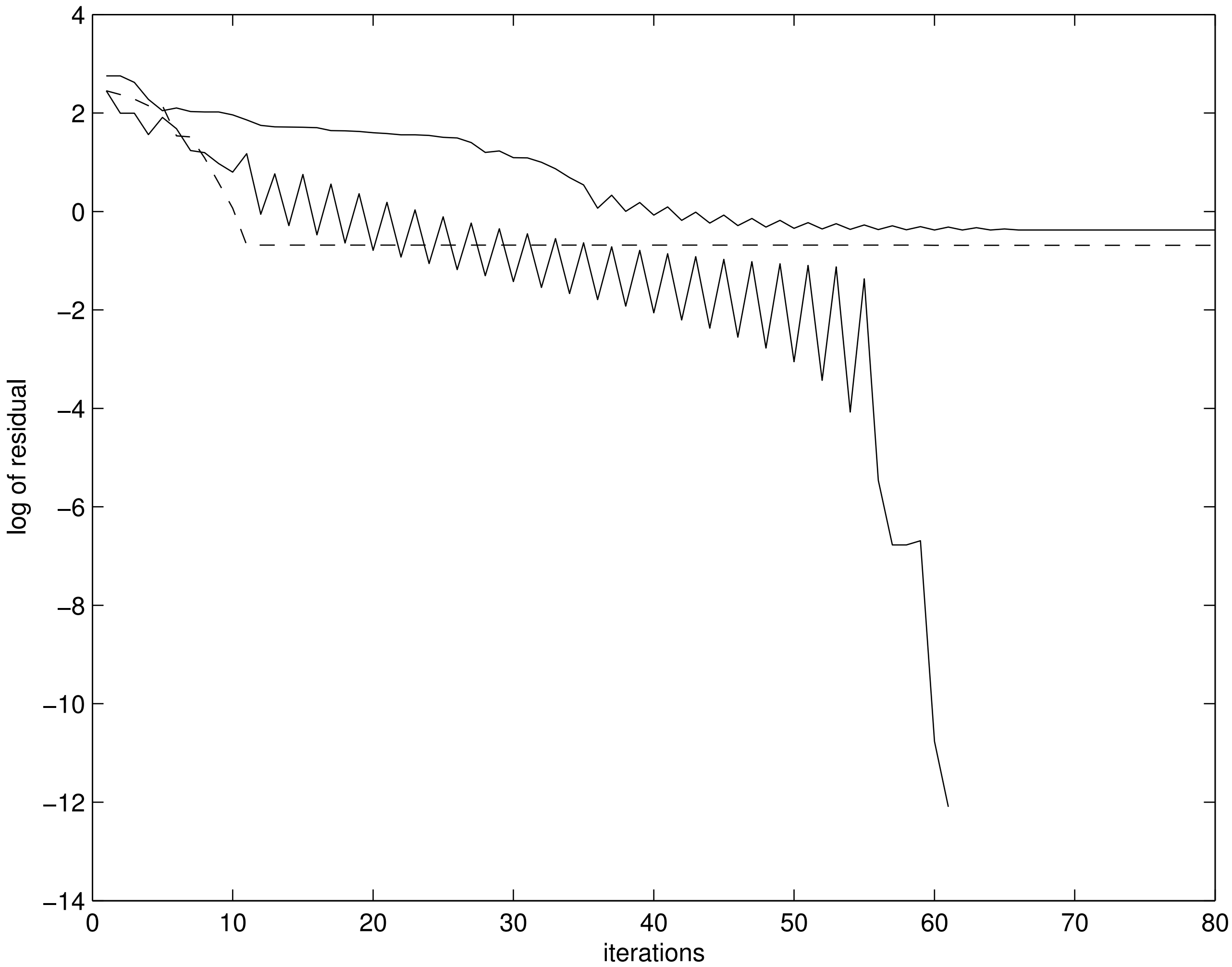


Filter vs. trust-region and LANCELOT B (iterations)

Plan
- Monotonicity
- Constrained opt.
- → ***Unconstrained opt.***

The University of Namur
www.fundp.ac.be

Numerical experience: HEART6

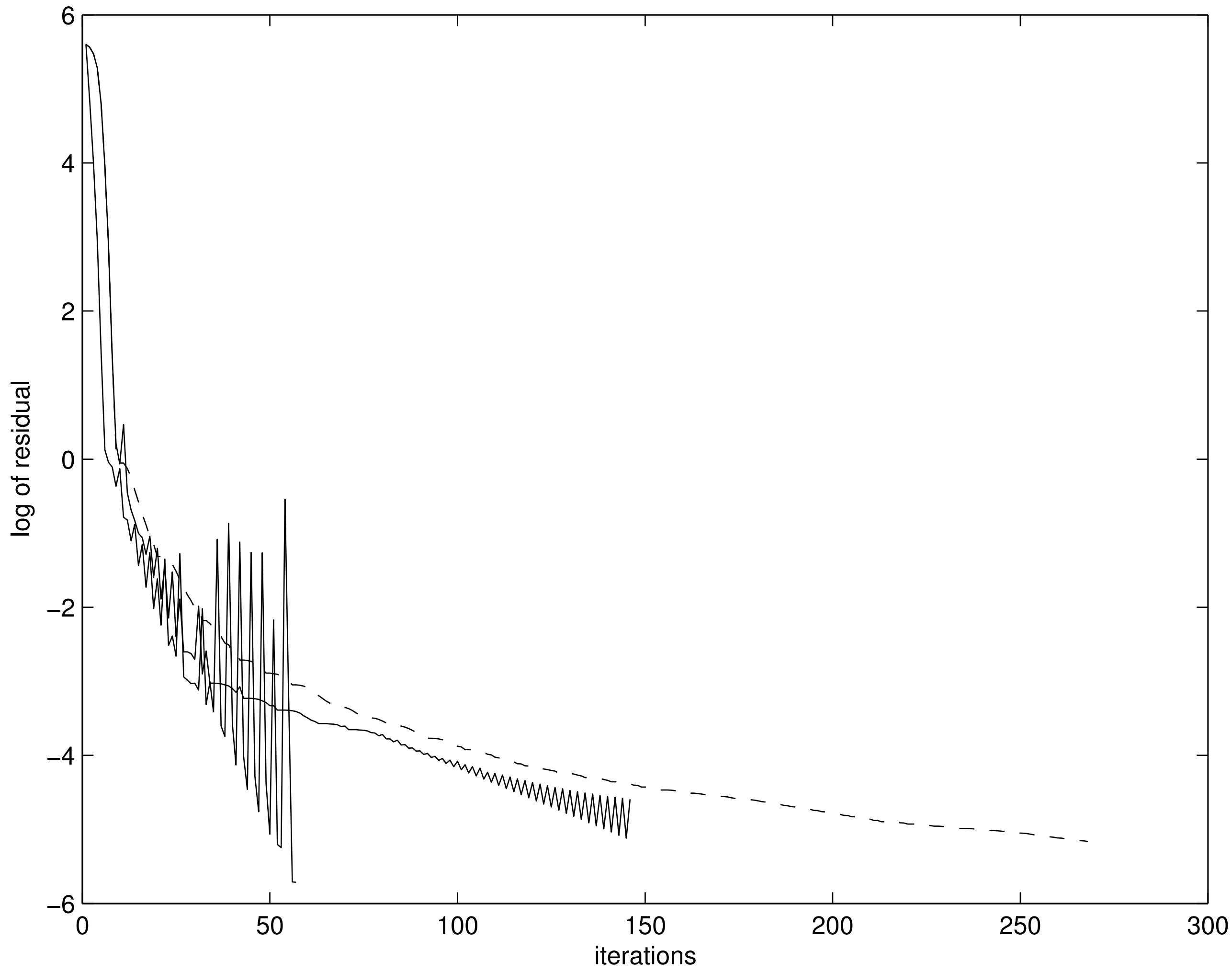


Filter vs. trust-region and LANCELOT B

Plan
- Monotonicity
- Constrained opt.

→ ***Unconstrained opt.***

Numerical experience: EXTROSNB

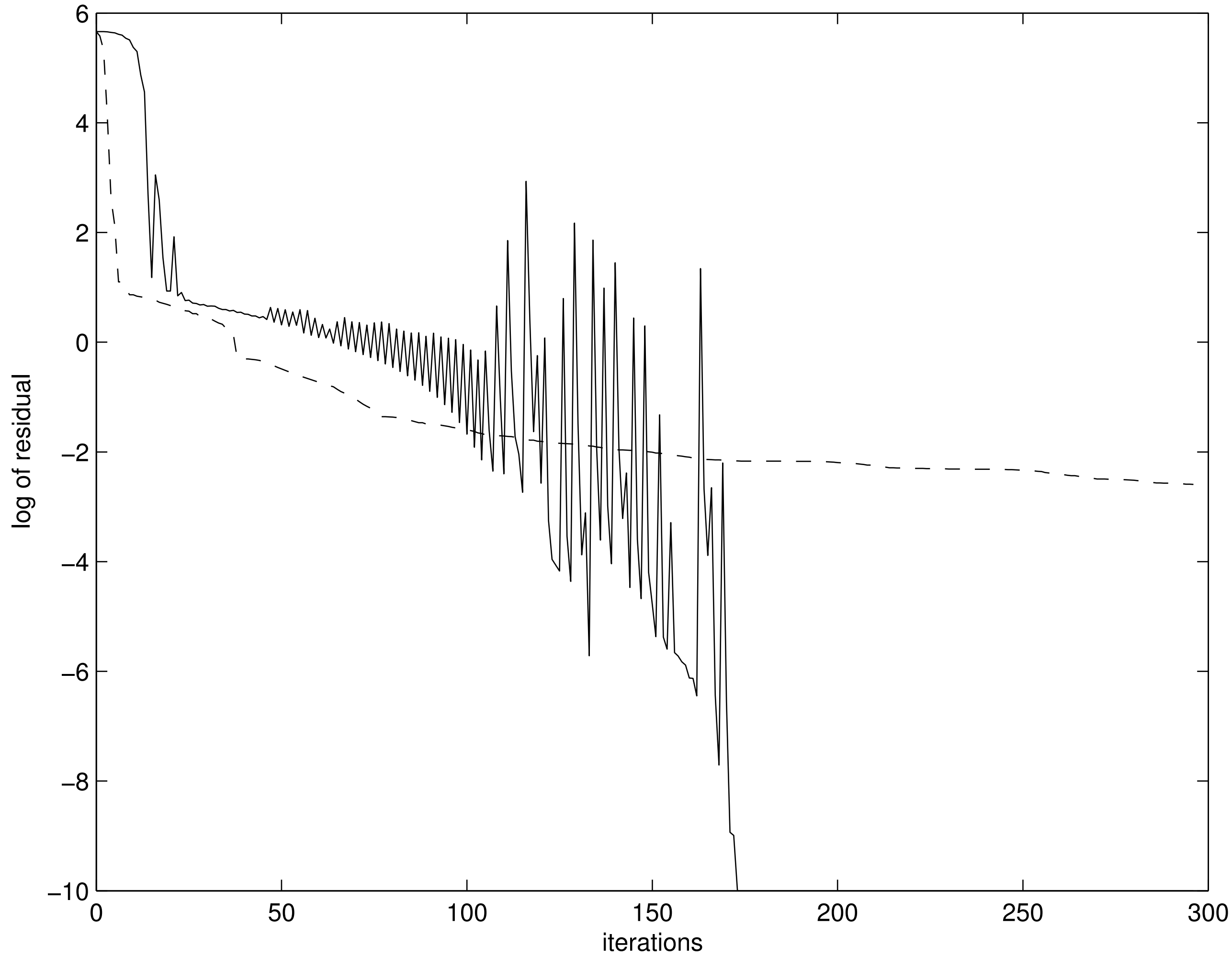


Filter vs. trust-region and LANCELOT B

Plan
- Monotonicity
- Constrained opt.

→ ***Unconstrained opt.***

Numerical experience: LOBSTERZ



Filter vs. trust-region

Conclusions

non-monotonicity definitely helpful

Newton's behaviour unexplained

. . . more research needed?

Thank you for your attention