Transportation modelling and operations research: a fruitful connection

Philippe L. Toint

Report 97/7 March 5, 1997

Transportation Research Group
Department of Mathematics
Facultés Universitaires ND de la Paix
B-5000 Namur, Belgium

This text is prepared as a support to the lectures given by the author in the framework of the NATO Advanced Studies Institute on "Operations Research and Decision Aid Methodologies in Traffic and Transportation Modelling", Balatonfured, Hungary, 10-22 March 1997.

Contents

1 Introduction

Although traffic modelling and analysis do correspond to the basic curiosity about the behaviour of fellow human beings, the true utility of these techniques really shows up when one realizes that they are absolutely necessary for defining transportation strategies at the level of today's society as a whole. And the necessity of such strategies can no longer be discussed today, given the staggering societal cost of the current transportation system.

- Around 55,000 people are killed every year on the roads on the countries of the European Communities only. A further 1.7 million are injured and 150,000 more are handicaped for life. The human and societal cost of these accidents just cannot be measured. Their purely financial cost exceeds 50 millions ECU per year.
- The financial cost of traffic itself is estimated to be around 500 billions ECU per year for the same countries, most of which is caused by congestion and inefficient route choice.
- Pollution caused by transportation (private vehicles, trains, busses, planes,...) is a serious concern. Its annual cost in Europe only is over 5 billions ECU.

These numbers, extracted from official UE documents [1] reveal the urgent need for a better planning of transportation. This urgence is further reinforced by the commonly accepted forecast that transportation demand is likely to grow at a sustained rate. For instance, the level of motorisation of peri-urbain dwellers in France has grown from around 4 to 6 cars for 10 adults between 1975 and 1990 (see [46]), and this growth is not likely to slow down in the coming years...

Understanding the transportation phenomenon in depth is therefore important. This understanding must ideally lead to tools (the transportation models) allowing traffic forecasts when the transportation network and/or demand and/or behaviour is modified. They first allow a better understanding of the process itself, but their principal use is for transportation *management*, both in its daily operation (traffic regulation), and for long term planning (strategic planning and investment policies). The many questions raised in these contexts are of course of extremely diverse nature. The group in which the author works has, for instance, studied the global impact of the displacement of a high school and an hospital on the traffic in a small city, the long term effects of ring roads, the efficiency of relief routes, the feasibility of automated urban transportation and the impact on congestion of variable message signs for incident warning, just to cite a few cases.

Quite naturally, transportation models are adapted to this diversity of application fields, but they all share a common purpose: that of making possible the establishment of strategies for improving comfort and safety of users, in view of political, economical and environmental constraints.

The acutest transportation problems arise of course in big cities. Most of these suffer from the negative effects of an extremely dense traffic and are indeed faced (to a varying degree) with all the nuisances caused by daily commuting. The basic traffic modelling methodologies naturally appeared in this context, but are the subject of a necessary and permanent re-evaluation. Indeed, the introduction of new information technologies in both urban and regional traffic has already modified the behaviour of many transportation network users in many cities. Frequent radio bulletins indicating congested areas and incidents are now part of the daily commuter routine, and have substantial impact on his/her transportation strategy. Further and more radical changes are afoot: route guidance techniques (autonomous or interactive), city map updating via radio beacons, variable message signs for traffic control and parking information, road pricing systems are but a few of the techniques studied and applied in the contemporary management of transportation networks. In Europe, international programs, as EUREKA PROMETHEUS and the DGVII and DGXIII efforts, have been set up to analyze and optimize the use of these new information and control technologies in a quickly evolving environment and society, but this line of research and development is also very active in the United States and Japan, where similar programs are proceeding. More generally, it is safe to say that the design and management of transportation systems today require to exploit every possible advanced and innovative technique with a potential role in this domain.

Clearly, operations research is a field which provides such techniques. It has been and still is one of the most active areas of development in mathematics over the past thirty years. Moreover, its object shares much of the preoccupations of transportation system analysts: the main idea is to provide a rigorous and constructive (algorithmic) analysis of a large number of management problems, of varying degree of abstraction, from the very practical application to the more theoretical aspects. The progress in this domain have been truly remarkable, especially in sectors covering both continuous and discrete optimization problems (see the collection of Handbooks published by North-Holland, and more particularly [52], for a global view of the field).

It is thus natural to attempt to explore the interface between transportation modelling and operations research and it is the purpose of this paper to point out some of the fruitful connections between these domains. This is not the first such enterprise (see [33], for instance) and the ambition is not of to be exhaustive, but rather to provide a broad framework in which the other workshop lectures can be situated and motivated, albeit in a relatively broad manner. The methodology chosen is to develop the main ideas of transportation modelling and to indicate, when suitable, the many connections with operations research. It would also be possible to proceed in the other direction, that is to provide links to transportation modelling from a development of the main ideas in operations research. We have chosen the first approach for its more practical nature and its more immediate motivation.

Section 2 considers, in a first stage, the context in which transportation modelling is used. Section 3 then covers some basic concepts in this domain, already providing some bridges with operations research. These bridges are even more apparent in Section 4, which is concerned with the description of the main methodological approaches. Some conclusions and perspectives are finally mentioned in Section 5.

2 The context of traffic modelling

2.1 Political evaluation

The first major arena where transportation modelling is widely used is that of the political evaluation of transportation policies at city, regional, national or even European level.

As alluded to in the introduction, transportation problems arise in the everyday life of most citizens. Any policy with a strong impact on the daily "way of life" of so many voters clearly warrants a careful evaluation from the political point of view. Furthermore, as the decision issues become more and more urgent and complex, decision makers are less and less willing to take strong options on the basis of their understanding alone and rely on (hide behind?) expert advice in a steadily larger degree.

Very importantly, the dialog between transportation modelling, as a technical field, and both the public and the decision makers requires a real professional skill. The lectures of Duchateau during the workshop will illustrate these points from the point of view of a practitioner.

2.2 Technological assessment

Transportation models are also used in technology assessment. Indeed, the technological development in transportation is often costly, which makes a sound economical evaluation of its effects and market crucial. The preoccupation is therefore different here from that in the previous paragraph: instead of considering the effects of transportation policies at a society-wide level, the impact of transportation related techniques is now considered from the point of view of specific actors (productors, implementation organizers) within the society.

The areas of transportation technique concerned by these considerations broadly fall in the following categories.

Infrastructural design. Probably the oldest field of application for transportation modelling is that of improvement and updating of transport infrastructures. Measuring the impact on traffic of network modifications, both in private and public transport, is indeed a very common application. The diversity of situations is nevertheless very large: motorways and urban roads, trains, busses, cars, cyclists, pedestrian routing, general infrastructure maintenance and all combinations thereof (multimodal transportation systems) provide a wide domain of investigation.

Technological advances in vehicles. A number of applications have been made in relation with today's technological advances in vehicles. Started many years ago with the "cruising" features (mostly in american cars), the innovations giving the vehicle some intelligence and more acute perception of its environment have flourished. Gap detection and warning, automatic speed adjustment, synchronized driving, all have their effect on the general traffic, not to mention the simple improvement in car performances.

Traffic management and control. Besides the problems posed by the availability to users of more sophisticate technology in their vehicle, the problems of traffic management and control also constitute a large class of applications where transportation modelling and associated methodologies play an important role: traffic lights settings, traffic control measures as speed enforcement or ramp metering on motorway access are just a few of the possible interesting topics in this category.

Technological advances in communications. Possibly the greatest challenge in today's modelling arises from the advent of advanced information systems for transportation users. These developments are based on the premice that informing transportation users modifies their behaviour. The behavioural modification, at the user's level, is then reckoned to cause significant change in the global transportation scene at city-wide level.

Of course, the problem is then to infer these global changes from the sometimes rather detailed knowledge of the information distributed. Questions of interest are then related to the broadness of information distribution, its accuracy, reliability, timeliness and method of transmission. The attitude of users with respect to this information is also a much studied subject.

Fleet management. The daily or strategic management of a private or public fleet of vehicles is also an important source of practical transportation modelling problems of true economic importance. They typically arise in freight distribution systems, routing of community services as garbage collection, school bussing, fire brigade or urgent medical care. They also arise in modern public transportation systems designed for handling variable demand by adapting their route definition, either on a daily basis on even in real time. Finally, such problems also arise in intelligent guidance systems, in particular those designed to bring the car users to free parking areas or to park-and-ride centres.

Regulations in the transport arena. Finally, the establishment of new regulations dealing with transportation also raises the question of their impact on traffic. One immediately thinks of parking policies, scheduling of goods delivery in city centers, road pricing or speed limits enforcement on motorways.

In many of these sectors, strong emphasis is put by the user on the reliability aspects of the involved models.

2.3 Land use strategies

We also stress here the intimate relationship between transportation planning and land-use regulations. Most transportation experts are very well aware of this relationship, but it remains a difficult task to integrate both approaches in coherent, practical and politically viable policies. The problems involved are the effect of transportation infrastructure on the localization of human activities and also that of human activities on travel demand and thus on transportation networks. This area provides a first connection between operations research and transportation modelling: the problem of optimizing the location of various activities, ranging from goods' distribution centres to hospitals, from manpower intensive industries to intermodal platforms, has been indeed widely studied in both fields (see [28] or [45], for instance). Optimal location problems are the subject of lectures by Labbé during the workshop.

Of particular interest are the interactions of the land use strategies and transportation systems in time. The dynamics of their interrelated evalutions is a fascinating subject, in which progress has already been obtained (see [65], [55], [56], for example), but where very active research is still ongoing (see [17] or [41]). The scope for applying operations research techniques in this direction is obviously very important.

2.4 Environmental analysis

A last domain of application for transportation models is that of environmental analysis and the complex question of designing transportation systems that are environmentally sustainable. The idea is that a number of nuisances generated by traffic are somehow proportional to traffic itself, and thus that traffic forecasts might help in predicting the evolution of these nuisances. Noise, gas emission and the related atmospheric modifications, vibrations and chemical effects of winter road salting are part of a set of preoccupations of growing importance in western societies.

This type of application is the subject of intensive study. It is complicated by the fact that the relation between pollution and traffic is often complex and not always fully understood: this leaves room for considerable debate and substantial methodological improvements. For some recent work, see [2] and other papers in the same volume.

An important question for a sound environmentally conscious transportation policy is that of generating cooperation between various agencies, regions or even countries on the effect of pollution generated by international traffic. This remains a very open subject, although possible approaches have been considered in [31] or [32] and the references therein.

3 The basics of transportation modelling

3.1 The spatial dimension

3.1.1 Zones and centroids

Like all models, transportation models considerably simplify reality. For instance, it is nearly always impossible to handle a level of detail where trips can be specified from building to building or from address to address in a city. It is thus customary to define *zones* that cover the considered area, the population and activity within these zones then being concentrated in a point, *the zone centroid.* Trips are then considered from an *origin zone* to a *destination zone.*

It is important to note at this stage that the definition of these zones is intrinsically tied with the availability of data. Indeed, if one wishes to estimate the transportation demand from or to one of the zones, one must clearly know some of their characteristics: population, sociological structure, attractivity for various purposes and accessibility, for example. It is therefore frequent that the very definition of the zones explicitly takes the availability of these informations into account, or, at least, the cost and effort necessary for obtaining them.

Besides the problems linked to data collection, other factors also influence the definition of zones. The purpose of the study being made may also suggest some natural aggregation. For instance, if one considers the traffic perturbations caused by the opening of a new shopping mall, it is clear that the immediate surroundings of the mail should be modelled with a certain amount of detail, while a more and more aggregate description might be adequate when the distance to the mall increases. Depending on the application, a city as Brussels could count from twenty zones to more than a thousand...

3.1.2 The transportation network

Once the definition of zones established, it is then possible to specify the transportation network itself, as the set of links between these zones. There will generally be more than one subnetwork, each corresponding to a different *transportation mode*: roads for cars, bus lines, cycle lanes etc. Such a complex network is then called a *multimodal* transportation network.

As for the zone's definition, different considerations influence the network description to be retained in the model and its level of detail. Availability of data about the network links, study purpose and integration of the network with the zone typically come into play. It is, in fact, rather fallacious to consider the definition of zones and the network specification as two successive and distinct phases: they are intimately linked to each other and should be integrated in a global and simultaneous approach.

The modelling of an urban area using these techniques is usually represented by a mathematical object called an *oriented graph*: *nodes* (points in the plane, for example) are linked together by "arrows" indicating the possible direction of travel (the *links*). Zones are then associated with nodes and network links to arcs.

3.1.3 Local or specific models

The above paragraphs focussed on modelling transportation at the level of a city or a region. But this is not the only type of modelling and much more local models are also of importance. The typical situation is when one desires to analyze, forecast or control a limited set of connected intersections. In this case, the definition of the zone under study often has nearly immediate implications in the definition of centroids and links: links are given by the actual roads and possible turns within the zone, while centroids and nodes are associated to the entry/exit points of the zones and with the inner intersections.

Modelling at other levels is also common when dealing with another part of the transportation system. For instance, techniques for modelling the transportation demand do not necessarily proceed by building an explicit representation of the transportation network, although there is still some concept of zones in this case.

3.1.4 New technologies and spatial modelling

The impact of new advanced technologies in transportation has an important consequence on the spatial description of models. The main observation is that the object of study is no longer restricted to the movements of users in the transportation network, but also covers the exchange of information across its various parts: sophisticate route guidance or incident detection as well as relatively classical traffic light synchronization involve that exchange very clearly. Furthermore, the efficiency of advanced transportation technologies is often said to be dependent on the quality, reliability, accuracy of the exchanged information, and on its distribution method.

Typical examples of relevant factors are listed below, where we attempt to indicate, in each case, the possible connections with operations research.

- A first factor is the location of traffic monitoring equipment (the actual locations of detection and measurement equipment on the network of course conditions the final information in that it specifies the parts of the network about which information could be available). The optimal localisation of this equipment (in order to cover the largest or most interesting part of the transportation system) may be viewed as a location problem on a network.

- The second factor is the location of information distribution mechanisms (conditioning in turn the locations at which re-routing can occur). Again, this can be considered as a location problem, either of continuous nature (a radio transmitter with limited range may be localized nearly anywhere) or discrete (variable message signs must be placed at specific locations on the network).

- Finally, one can consider the actual interconnection pattern (topology) of monitoring devices, information processing centres and distribution mechanisms. This is a network design problem, a very widely studied class of combinatorial optimization applications.

Besides the classical transportation network, we therefore discover another network, which describes the information flow (the feedback loop) between the transportation network and its users. We call this network the *information network*. It is the author's view that explicit modelling of this information network is an important part of the present research challenges in this area. Of course, these information network models should be flexible, in order to adapt to the many different cases of potential interest. In particular, they should include the relevant factors mentioned above, but also parameters that influence their efficiency (communication links' or detectors' reliabilities, for instance) and thus also permit the modelling of information system breakdowns.

3.2 The time dimension

A fundamental distinction appears at this stage between two distinct model classes: *static* and *dynamic* models. As could be inferred from these terms, both approaches differ by the way in which they consider the traffic evolution in time.

Historically the first, *static models* describe traffic on a time period that is long enough to ensure that all travellers starting a trip in this period also reach their destination within the same period.

One therefore obtains a considerable conceptual simplification, because it is no longer necessary to analyze traffic on the network links at successive moments. It is indeed sufficient to consider the cumulative value on the considered time period. This simplification is of course made at the expense of some realism, but the results obtained with this approach can however be adequate for some kinds of studies. (As the wear of road surface is roughly proportional to the total volume of traffic on the road, but is independent of its repartition in time, it is thus enough to know an approximation to this total volume for establishing a maintenance program for the city roads...).

Dynamic models explicitly introduce the passing of time within their mechanism. These models originate from the deficiency of static models in correctly describing route choice in peak period, where the traffic on the network links can significantly differ from its average value on the day, say. Moreover, not only the routes but also transportation modes and departure times can vary: dynamic models are thus *necessary* to describe transient traffic phenomena, as congestion experienced by commuters on the big cities' approaches.

Practically, these models are much harder to build because the quantity of needed parameters and results is usually much larger than for static ones. It is indeed necessary to make every quantity fully time dependent. The requested data collection as well as the a posteriori validation are therefore considerably more complex.

As most models for time dependent phenomena, dynamic traffic models can be classified into three broad categories, depending on the method they use for representing the passing of time.

- Continuous models, first, consider traffic parameters as continuous functions of time, and they aim at determining some of these functions (as traffic density, for instance) as a consequence of other such functions. It is in this context that Lighthill and Whitnam developed in 1955 [44] their famous model for a phenomenon well known by many road users: the "moving traffic shock waves" delimited by zones of slowing and zones of accelerating traffic.

- The discretized models, on the other hand, describe the time dependence of traffic parameters by representing the time in successive "slices" during which the transportation network has a constant behaviour. The studied effects (as congestion) therefore vary from one timeslice to the next. They are often calculated from laws that link their value at a given timeslice with other values from preceding timeslices.

This technique has the further advantage to allow a progressive transition between static and dynamic models. A static model can indeed be considered as a dynamic model with only one (long) timeslice. The model is then made "dynamic" by increasing the number of timeslices and reducing their length. As a result, one obtains mixed models whose behaviour is similar to that of static ones for long timeslices (from 0 to 6am, for instance) but still allow for a finer time analysis at peak periods, when timeslices are shorter.

- A third way to handle the passing of time in models is the so-called "event driven" technique. Time is considered as a sequence of events, each of them implying some modification in the state of the model, with the possibility for an event to define one or more other events in its future. Such events might be traffic lights changes, arrival of vehicles at intersections or bus-stops, a traveler's departure or arrival, network incidents or information transmission. This technique is thus especially adapted in the treatment of transient phenomena, since these are mostly caused by such "events" in the network, and to models involving some type of incident detection and warning.

  Another advantage of this methodology is that it is most suited to exploiting parallel computer architecture, whenever available. Different events in different geographical areas can indeed be handled in a completely asynchronous fashion by different processors. This feature is far from being anecdotic, because the computational requirements of dynamic models is in general substantial, and techniques to speed processing up are very worthwhile in the framework of real time applications.

The distinction between continuous and discretized models is further blurred in practice because continuous models often use numerical techniques that rely on an internal time discretization: timeslicing is thus implicitly present even in purely continuous approaches. Similarly, dynamic models with time-slices may also be considered as "event-driven", where the only events are associated with the end of each time-slice.

Because much of advanced technologies in the transportation arena are geared at the dynamic behaviour of transportation users and at the control or analysis of transient phenomenons, dynamic transportation models seem unavoidable within this context. Network users indeed have clearly different strategies in peak hour as opposed to off-peak, in week-ends as opposed to week days, in day as opposed to night, and the list of examples can easily be made longer. Furthermore, techniques like intelligent incident detection and interactive or automated route guidance are only meaningful in a dynamic context. It is remarkable that, although appreciated by many experts, this conclusion has not always been applied in its full requirements.

We conclude this paragraph by noting that the difficulty of evolving from a static to a dynamic model varies very much with the specific area of transportation considered. If dynamic route choice models are becoming "of age", the transition is much more difficult in demand analysis or land-use interactions, for instance.

3.3 A paradigm for city-wide modelling: the four stages model

Once the decisions about the territory and time representation taken, the model is already well shaped: it contains zones and a network for travelling from one zone to another. We should then use this structure for describing the traffic itself. We now focus our attention to a very classical methodology for city- or region-wide modelling originally proposed by Wilson in the beginning of the seventies [68]. This method proceeds in four successive stages:

Traffic generation. In the first stage, each centroid of the model is considered and the number of trips leaving the associated zone, a well as the number of trips ending in the zone are determined. This is the *generation stage.*

This stage classically uses statistical techniques for predicting the "emissivity" of a zone, that is the number of trips that depart from that zone. These techniques are based on the repartition of inhabitants (households) in coherent socio-economical groups, whose displacement behaviour can be analyzed. Similarly, the "attractivity" of a zone, i.e. the number of trips ending in the zone, can be established for various causes of displacement (the *functions*: work, shopping, school, etc.). Data is typically obtained from special surveys.

Traffic distribution. The second stage then consists in calculating the repartition of these trips in (origin, destination) pairs. One speaks of the *distribution stage*, whose result can be placed in a big table: each entry of the table contains a number of trips leaving the zone corresponding to the row of the table and ending in the zone corresponding to its column. This is the well-known *origin/destination matrix*, or O/D matrix.

Modal choice. The third stage models the competition between the various transportation modes (private cars, train, busses, metro, cycles,...) and establishes, for each (origin, destination) pair, the proportion of the trips using the different modes.

Traffic assignment. Finally, the fourth stage, called the *assignment stage*, determines the actual routes in the network that are being used for each of the trips and each of the modes during the considered modelling period.

This four stage methodology has often been criticized because of its sequential character. For example, the choice between car and bus for a trip may depend on the congestion level on the roads or in the public transportation system, level which is not known until after the fourth stage. Congestion can also encourage drivers to change their destination, or even, in extreme cases, to forego their trip altogether. More generally, the four *successive* stages are often integrated and interdependent in practice.

We note that we haven't specified typical methodologies for the last three of the four stages. A survey of these methodologies is the object of the next section.

We finally note that the four stage methodology can also be applied, even partly, in more local studies, like the analysis of a set of intersections. In such a case, one would typically estimate the O/D matrix from measurements, say, which correspond to Stage 2, and then assign the corresponding demand on the considered network, which correspond to Stage 4.

Unfortunately, using the four stage modelling approach is not always a reasonable methodology. For instance, if the problem under consideration contains user information systems, the inherent sequentiality of the technique is then completely unrealistic, because information flow from one of the four stages to previous ones is very common in this context. For example, home-based information systems on expected travel time using private or public transportation are designed to modify the modal choice (Stage 3) as a function of network performance (Stage 4). Congestion dependent road pricing schemes have a similar effect, but are also anticipated to alter the transportation demand (Stages 1 and 2) itself.

Although a number of dynamic models have been proposed for traffic assignment (Stage 4), very much less is known about dynamic city-wide demand models, and even less on the necessary integration of both. This is an area where "classical" transportation modelling truly needs improvement. The situation is less critical for more local studies, as there exist methodologies for estimating a dynamic demand in this case (see Section 4.6 below).

4 Methods in transportation modelling and their connections with Operations Research

We now examine some classical methods and concepts whose application in transportation modelling is very widespread. Again, we have no ambition to cover the field: the reader is referred to the excellent encyclopaedia [53] or to [8] for further analysis.

4.1 User's rationality and utility functions

One of the most important question in modelling human behaviour is to decide on what is considered to be the "motor" or driving force behind the choice of a particular action. This very central question will probably always be open for debate. However, practical models have to make some kind of choice at this level. The main dimension along which existing methodologies can be ranked is the assumption that transportation users make, at least to some degree, *rational choices*. It is commonly assumed that using a transportation network always implies a certain cost for the user (economists talk about "disutility"): not only travelling takes time, but it could also cost in several other ways. The ticket price for the bus or the metro, the petrol consumed by our vehicle, the number of stops along the route, the accident risks, the inconvenience and stress caused by congestion... can also add to the mere time spent to our perception of transportation costs. Society as a whole also supports other costs associated with transport, as pollution, security, insurances, medical care, noise and other nuisances caused by traffic. As a result, many transportation modelling techniques take the point of view that these costs should be minimized, which is a classical linear or, more commonly, nonlinear optimization problem. Of course, the problem may also involve constraints (such as satisfying demand or not loosing travellers on the network). In practical instances, the problem also depends on how exactly the costs are defined.

The notion of global cost associated with displacement is typically taken into account by the choice of a *cost function* or *performance function* that determines, for each link of the network, a "cost" as a function of the traffic on that link. Units for this cost vary from model to model, and are chosen in relation with the final purpose of the modelling. For example, the quantity of exhaust gas of a certain type can be used if one wishes to measure the pollution impacts, while transportation time can be used if one is interested at asserting the level of service of public transport... There is however a remarkable consistency between all these choices, irrespective of the units chosen: the cost function is typically increasing as traffic along the link increases, slowly first but faster and faster for higher traffic levels. For example, the travel time along a link is relatively independent of traffic for low traffic levels, but can increase substantially for more congested levels, up to the point where not a single vehicle can move because everybody is stuck in a traffic jam!

It should be noted that the choice of a single performance function associated with the parts of the transportation network implies a complete and uniform rationality of the users in the measure where each users minimizes his or her disutility. This assumption has often been criticized as irrealistic. One way to overcome the difficulty is to introduce some degree of stochasticity in the users' choice criteria. We will briefly discuss these aspects below.

Finally, optimizing a single "utility function" (even containing some stochastic component) often raises the question of comparing various goods or services of qualitatively different nature, a very difficult task. For instance, it is often necessary to estimate the monetary value of time (VOT) for users or users' classes. Some concepts in multi-objective optimization constitute an interesting alternative at this level (see [22], [42] or several papers in [8] for an example) and will be developed further in the course of the workshop by Tanczos.

4.2 Physical analogies

The first generation of concepts in transportation modelling is widely based on analogies with the physical world. Traffic is considered as a flow (see above) or an electrical current. Various methods have been proposed to exploit these analogies. We will briefly describe here two "physical" concepts that are still of major importance in methods for traffic distribution, the second stage of the classical four stage model.

The first idea is that transportation demand between two centroids might be modelled using the law of *gravitation*, in which the intensity of the attraction force between two bodies is proportional to the masses of these bodies and inversely proportional to the square of the distance separating them. Transposed in transportation demand models, the trip demand from a given centroid i to another j is then proportional to e_i, the "emissivity" of i, and to a_j, the "attractivity" of j, and also proportional to some decreasing "deterrence" function f of their (shortest) distance d_{ij}. More formally, the number of trips q_{ij} between centroids i and j is then approximated by

$$t_{ij} = k\ e_i a_j f(d_{ij}). \tag{1}$$

In the strict gravitational framework, one chooses $f(d_{ij}) = 1/d_{ij}^2$.

Another somewhat similar idea is to use the concept of *entropy* in a physical system. In this approach, the distribution model attempts to recover a particular trip distribution such that its likelihood is maximized amongst all equiprobable demand configurations. An easy mathematical development then shows that

$$t_{ij} = k\ e_i a_j e^{-\sigma d_{ij}}, \tag{2}$$

where σ is a free model parameter. We immediately note the strong link between the entropy and the gravitational approach (see [59] or [9], for instance, for a discussion of this relationship). One then typically distinguishes (see [60]) between "singly constrained" distribution models, where the emissivity of all centroids is assumed to be known, and "doubly constrained" models, where constraints are put on both emissivities and attractivities. The choice of a suitable model for traffic distribution might be a delicate question (see [60]). For instance, it is often reckoned that singly constrained models are more appropriate for short-term studies or for studies involving non-work functions.

Needless to say, these physical analogies typically lead to the solution of optimization problems. Entropy is maximized and one performs a fit for a gravitational model. The use of operations research is thus again present.

We finally note that the use of these concepts in a dynamical context is somewhat difficult.

4.3 The assignment of traffic

We now turn our attention to some more mathematical techniques that are mostly used within the fourth stage of the general model described above, that is the assignement of traffic within each modal network, given the demand.

4.3.1 Shortest paths

The first popular idea is that network users choose the route that is *shortest* (in time or in any more general cost measure), while assuming that the cost associated with each link is constant, i.e. independent of the traffic on that link. Finding such a shortest path is maybe one of the oldest operations research problems. It has been widely studied. The reader is referred to [61] or [3] for further details and references.

This technique has the advantage of being readily interpretable and also extremely efficient from the computational point of view. One indeed knows very efficient algorithms for computing shortest paths in a network, even when the number of nodes and links is very large (over 100000 nodes is quite feasible). Traffic assignment based on this idea is known as the *all-or-nothing* assignment, because all trips between a given (origin, destination) pair are assigned to the same single route, a rather crude approximation.

Although mostly used in the static context, dynamic versions of shortest paths algorithm do exist (see [22] for instance). Their efficiency is considerably less than their static counterparts, but again the growth in computer power makes them more and more attractive for practical use.

Along the same line of thought, one may also consider the question of reconstructing the costs associated with routes in an unsaturated transportation network, as they are perceived by the network users. This is important if one wishes to take perception (in some ways a form of behaviour) into account. A first approach has been proposed and tested in [14]: the idea is to reconstruct the perceived delay associated with each link of the network from the observation of the paths actually taken (assuming that users choose the shortest route between their origin and their destination). This method, which in fact solves the *inverse* shortest path problem, is akin to the idea of using "mental maps" [24] in the process of route planning. Technically speaking, the network under study is represented by an oriented graph in which a set of shortest paths is known; the question is then to infer the value of the time delays associated with each arc of the graph and differing as little as possible from a set of a priori known costs (derived, for instance, from the knowledge of the geometrical characteristics of the road). Applying this methodology to urban situations, it is important to explicitly consider the delays at signalized junctions, and not to restrict the analysis to the estimation of the delays on the links only. This can be achieved by using a graph that contains detailed arcs to represent the various "turns" in a junction. The resulting problems, where the unknown costs on these turns are correlated (for instance by traffic light phasing), can also be solved reasonably efficiently, as shown in [15].

4.3.2 Wardropian traffic equilibrium

Probably the most famous and most commonly used assignment method is aimed at improving this approximation by assigning trips to more than one route between a given (origin, destination) pair. It is based on the notion of *traffic equilibrium*. Following the famous statement made by Wardrop in 1952 [64], "travel time on all used routes is smaller or equal to that that would be experienced by a single vehicle on any unused route". In other words, no network user can, in an equilibrium situation, improve its trip cost by a unilateral decision. One easily sees that a consequence of this assumption is that all used routes between an origin and a destination have the same (minimum) cost and that all routes of higher costs are unused. This concept is obviously substantially more complex because it implies that the choice of a route between two zones depends not only on the possible routes between these two zones, but also on the load of the network, resulting from the traffic between all other O/D pairs! This concept is called "User Equilibrium" because the criterion used is the travel cost for the network user. The work of Beckman, McGuire and Winston [5] was amongst the first to use this principle in a very long line of research (see [19], [26], [27], [30], [58], [60] or [54], for further discussion and references). It is important to note here that a number of the most efficient methods to compute the User Equilibrium are based on the observation that *the equilibrium conditions are in fact the optimality conditions of a well defined continuous linearly constrained optimization problem*, providing yet another link with operations research. This crucial link is very well developed in [60] and [54].

One can also consider the "System Equilibrium" which would occur if one chooses all routes to minimize the total transportation cost for all users (or for society). These two equilibrium concepts are unfortunately different, because the advantage of all may require that some users deliberately choose a route that is less favourable for them... The (alas) unrealistic nature of this assumption implies that User Equilibrium is most often used for describing the behaviour of users in the network, while the System Equilibrium is sometimes computed to obtain an "ideal" traffic situation.

The difference between User and System Equilibrium is best illustrated by the famous Braess paradox (see [13], [29], [21], [34], ...). This example shows that, in the presence of congestion, the building of a new road may result in a deterioration of the network performance, if one assumes that each user optimizes its own travel time. Generally speaking, it indicates the importance of an in depth analysis of planned changes in the transportation network. It also shows, a contrario, that traffic restrictions (like pedestrian zones, one-way streets or road pricing) can have a beneficial impact on the global situation.

4.3.3 Equilibrium and variational inequalities

It is important to note that the equilibrium method has been extended to handle situations more general than pure traffic assignment: one can talk of "equilibrium" between various transportation modes or of "equilibrium" between transportation demand and transportation supply (as specified by the network). The first of these extensions typically requires a modification of the equilibrium concept, because the cost of public transportation cannot in general be compared to that of using a private car. Similarly, entropy ideas are usually introduced in the second extension for improved realism. We observe that both these approaches answer, albeit in part, to the criticism on the excessive sequentiality of the four stages model, because two or more phases are now interdependent and solved for simultaneously. The reader is referred to [60], for instance, for a further discussion of the many extensions of the equilibrium methodology.

From the point of view of the connections with operations research, one of the most interesting extensions of the equilibrium concept is to problems where the interactions between parts of the model ruin the immediate translation of the equilibrium calculation into an optimization problem. This happens for instance if several classes of users are considered on the network, or if there are interactions between the flows on different arcs. One then focusses more directly on the equilibrium conditions, which can be considered as a particular case of *variational inequalities*. The numerical treatment of variational inequalities in this context has also been an area of research in between operations research and transportation modelling, and is the object of specific lectures by Ferris during the workshop. One can also refer the reader to the work by Dafermos and followers, [19], [20], [50], [51], or to [54] or [63].

4.3.4 Stochastic models

Both complete equilibrium assignment and the simple shortest paths technique assume that all network user see the cost of their route in a uniform way. The variability inherent to human decision processes has very little room here, which can be judged unrealistic (see [35], for instance). As an answer to this criticism, several stochastic variants of shortest path techniques have been proposed with the aim of re-introducing some variability while maintaining the assumption that the cost associated with a link is independent of traffic: the idea is to assign a random utility to "reasonable routes" or even to individuals links in the network.

The first of these techniques, introduced by researchers like Dial around 1970 [23], consists in finding the set of reasonable routes between an origin and a destination. A route is said to be reasonable in this context if the users steadily increases its distance from the origin and decreases its distance to the destination when following the route. The shortest route is of course reasonable, but the set of reasonable routes is typically larger. The traffic is then spread over these routes, using a logit distribution function. In another approach, one typically averages the flows obtained on a given link during successive applications of the shortest path assignment method applied in a network whose link costs are drawn from a normal distribution with fixed mean. This "successive average" method correspond to applying the probit technique to the route choice problem.

Similarly, one can introduce the notion of stochastic user equilibrium (SUE), in which each users minimizes its own *perceived* travel time in the network, where the perceived travel time is assumed to be a stochastic perturbation (of known distribution) of the real (measured) travel time. A good introduction to this technique is provided in [60], for instance. We briefly summarize the relations between assignment methods in Table 1.

We note that the stochastic nature of travel time can be interpreted in two ways. The first and more traditional interpretation is that the random perturbation of travel time represents the variation of perceived travel times with respect to real ones, induced by the variation of perception from a user to the next. Since these perceptual variations are essentially unavailable for measurement[1], they are assumed to be random. The second interpretation is that the user itself assigns a random component to travel time, for instance to represent aleas as waiting times at signalized intersections and variations in travel speed due to weather conditions or other random events on the network. In this second

[1] See the comment on inverse shortest path in Section 4.3.1 for a possible approach.

	constant costs	traffic dependent costs
deterministic approach	all-or-nothing	equilibrium (UE)
stochastic approach	reasonable paths or averaging	stochastic equilibrium (SUE)

Table 1: Relations between some traffic assignment techniques

interpretation, the user is not trying to optimize its travel time as he perceives it, but is instead trying to take into account random variations of network performance, or, more generally, of the environment, that are beyond his or her control. This attitude is typical of the vast class of operations problems known under the name of stochastic programming. For instance, one may also consider the problem of traffic assignment with stochastic demand... Stochastic models in transportation will be the subject of the lectures by Louveaux during the workshop. Reference books include [25] and [39].

4.3.5 Dynamic traffic modelling

For the reasons discussed in Section 3.2, one may wish to include the time dimension in a traffic assignment model. This effectively comes in three main flavors.

The first approach is to generalize the concept of equilibrium to time-dependent equilibrium. The first steps in this direction were done by Merchant and Nemhauser in [48] and [49] and were followed by a long line of contributions. See [69], [70], [66], [67], or the recent book by Ran and Boyce [57] and the references therein, for instance. A debate remains amongst transportation modellers on the validity of the dynamic equilibrium concept, mostly because the assumptions of full information and full rationality, which are necessary in this context, may be judged even more unrealistic than in the static case. Models of this type are often of the continuous type and are based on the classical concepts of optimality conditions in nonlinear programming or variational inequalities, two branches of operations research that we have already mentioned.

The second approach attempts to replace the definition of an equilibrium situation by a simulation of the evolution of traffic over time, without any reference to a "steady-state". Traffic is then considered as a pure dynamical system[2].

[2]In the sense of the mathematical theory of dynamical systems: the way the model behaves

The difficulty is then to define what replaces the concept of equilibrium in the mechanism for specifying route choice for the network users. One may again try to minimize some variant of travel time (subject to the available knowledge of the network conditions) or one may define a more general behavioural theory for users, as is done for instance in the PACSIM model discussed by Toint in another lecture of the workshop. A good reference for the questions raised in this context is [6]. Simulation models are often of the discretized type. The subject will be discussed in the workshop by Barcelo and Mahmassani. Because simulation is another classical subject in operations research, models of this type provide yet another link to this field.

Finally, one may consider dynamic traffic models of networks less complex than a complete city. This is especially useful in traffic control, where one tries to optimize traffic in a complex intersection, or to control the access to a motorway in order to avoid saturation and the resulting delays. Questions of this type will be discussed by Papageorgiou during the workshop.

4.4 Routing and related problems

Another important question in transportation modelling is often to compute routes for vehicles or users subject to constraints more complicated than those present in the simple assignment problem. These may include constraints on goods delivery, passenger transfer, sequence of visites sites, time windows or vehicle capacity, just to cite a few of the most commonly occuring cases. The determination of a route satisfying all constraints then become another classical problem of operations research and combinatorial optimization, the routing problem. It has applications as diverse and as pervasive as school bus routing, post delivery, garbage collection, road maintenance planning, demand sensitive public transportation systems, or "just in time" freight delivery. Problem of a similar combinatorial nature also arise in other contexts related to transport, as train time-tabling or traffic light setting. As can be seen from these examples, the applications of combinatorial optimization in the transport arena are plenty. The interested reader is referred to [8] for a nice collection of recent contributions in this area, and also to the lectures of Laporte and Toth during the workshop.

4.5 Discrete choice techniques

Another powerful tool for transportation modellers is the theory of *discrete choices*. The main idea here is to assume that transportation users, when faced with a decision, can only choose between a limited set of possibilities, hence the name of discrete choice. The typical situation is that of modal choice (stage 3 in the four stages model), where this set might include private car, public transportation, cycling or walking, for instance. With each of these possible choices is associated a utility function, and it is assumed (again) that the user maximizes his utility (another operations research problem).

The difficulty arises from the fact that these utility functions are usually not accurately known and cannot be measured easily. Moreover, the factors

at a given step only depends on the previous steps of its history.

(attributes) that influence these utility functions (such as individual variability in sociological status, history, etc) are also beyond a complete analysis. To circumvent this problem, the utility functions are then constructed such that they contain a random part, which is supposed to represent the contribution of all these unknown factors (see our discussion of the stochastic user equilibrium assignment). One then talks about *random utilities*. A random utility then merely gives the probability that a given choice will be made by the considered user.

The first major discrete choice model is the *logit model*, in which the probability of chosing a given alternative k among K possibilities is given by

$$p_k = \frac{e^{d(k)}}{\sum_{i=1}^{K} e^{d(i)}}, \tag{3}$$

where $d(i)$ is the deterministic part of the utility associated with the i-th choice[3]. As can be seen from this formula, this (sometimes irrealistically) assumes independence of the possible choices. The logit model is very widely used in all stages of transportation models: the modal choice is often represented by a logit model and the choice of reasonable routes (see below) can be made using this approach as well. The popularity of the logit model is strengthened by its ease of use in multinomial cases, that is when the number of possible choices for the user exceeds two. In this case, one often "cascades" the choices as a sequence of successive decisions, each of which being modelled by (3). One then talks about *nested logit* modelling. The definition of these models may involve some problems of their own, that are connected to operations research in a loose way (see [11]).

An alternative model, the *probit* model, is based on the more general assumption that the utility functions are normally distributed but not necessarily independent. This model is however more difficult to implement because it leads to difficult numerical questions and also requires additional information on the choices (the covariance matrix), compared to the logit approach.

The reader is referred to [7] for a comprehensive coverage of discrete choice theory in the framework of transportation modelling. This subject will be covered by the lectures of Bierlaire during the workshop.

4.6 Activity chains

We now briefly introduce another methodology, *activity chain* modelling, which is related with discrete choice models in that it attempt to capture the behaviour of transporation users. One of its main objectives is to help in the development of inherently dynamic and behaviourally sensitive demand analysis tools.

The notion of an activity chain is nearly self explaining: this is the sequence of activities planned for one or more individuals of a household at a given time in the day and week. Given such chains (the more one knows the better), it is

[3]This formula is based on the assumption that the random terms in each utility function are independently and identically distributed Gumbel variables. The Gumbel distribution is an analytic approximation of the normal distribution.

then possible to deduce the transportation demand by considering which successive activities do not take place at the same localisation. The examination of many such activity chains then leads to an intrinsically dynamic and behaviour orientated demand modelling, resulting in more classical dynamic O/D matrices. The advantages of this approach are mainly

- its intrinsic dynamic nature,
- its very adaptable framework (it covers, for instance, multipurpose trips, a common reality but gaping hole in many other modelling approaches),
- its sensitivity to behavioural variations across households, because different schedulings of the same activities are possible.

Of course, this technique is not without its drawbacks.

- It requires a large amount of data (travel diaries, mostly) which is often difficult to obtain.
- The structure of the activity chains is not always known a priori, which introduces yet another level of modelling, that of activity scheduling.
- Its practical treatment by reliable software tools is still in infancy.

Despite these serious inconvenients, activity chain modelling remains one of the most promising techniques, especially when one wishes to take the behaviour of the users into account, such as in models of information systems. References in this area include [37], [4], [36], [62] and [47]. The subject will be touched by Polak in the course of the workshop.

In the context of activity based modelling techniques, another component of the traditional operations research scene plays an important role. As activities of a household or individual happen in time, there is a clear need for scheduling techniques in the establishment of the activity chains themselves. However, the author is not aware of many research contributions aimed specifically at bridging the gap between activity chain models and scheduling techniques.

4.7 Other continuous optimization and estimation methods

We close this overview of the methodologies used in transportation research by mentioning that, as is the case in many scientific applications, one often tries to fit a given model to a set of observations. For instance, one might try to fit a OD-matrix describing the transportation demand to observations of traffic flows or centroids' emissivities. Or one might wish to calibrate an assignment model to adequately represent the observed network flows. This calibration is usually carried out by making the model's output as close as possible to otherwise known measures of the real situation[4], a classical parameter fitting problem in operations research.

[4]It is therefore important for a modeller not to use all the information he has at his disposal to build the model, but to keep some of it for this calibration phase.

Once calibrated on the real situation (often called the *reference case* or the *do-nothing case*), the model can then be used to analyze diverse alternative scenarii, differing from the reference case in a well defined manner. The confidence one has in the obtained results for these alternative scenarii is clearly conditioned by the quality of the model calibration on the reference case, and thus, albeit indirectly, by the quality of the method used in the calibration phase.

Among the many methods for observation fitting, the least squares approach is often preferred (see [16] or [12] for an example of application to the distribution stage). One should also mention here that some inherently dynamic methods for local demand analysis can also be viewed in the framework of the least squares approach, in that they try to exploit correlations between flows penetrating into and leaving the study zone (see [18], [40] for instance).

Other optimization problems arise in data calibration. For instance, the parameters of a (nested) logit model are typically computed using a maximum likelihood estimator, itself resulting in a specific unconstrained continuous optimization problem (see [10]). And the list is far from complete...

5 Some limits of transportation modelling as an assessment tool

In this section, we briefly discuss some limitations that should be kept in mind when using transportation models as an assessment tool.

The first consideration is that a model can only really be used if it has been validated in real applications, and when its behaviour has been shown to be robust enough to allow reasonably safe extrapolation. While this is often the case for classical models, this is much more rarely the case for newer and more advanced models (as can of course be expected). This remark is especially true for dynamic models, where validation is a much harder task. Hence the necessity of pilot experiments and associated model appraisal. The "Traffic Appraisal Manual" of the english Department of Transport (1994) or the work by Leurent in his thesis [43] attempt to set up "rules of good practice" in this domain, but much remains to be done. One could mention at this point the conclusion drawn by Julien and Morellet in [38]: "Finally, the real debate on the choice of the type of modelling should not deal with what these or those models can do theoretically, but rather with the more or less important hazards to which the calibration of these models can be subjected". Although the author feels that this conclusion is probably too severe, it also has its part of truth...

Correlative to the validation question is that of data collection. The more complex the model, the more data it typically requires, hence the more difficult to apply it with its full predictive power. This limitation has to be recognized, but does not necessarily prevent a cognitive use of the models as a tool to gain intuition and understanding of a practical situation. In particular the collection of data related to transportation user behaviour is a challenging task. This question is partly covered by the lectures of Polak during the workshop.

A final and insidious limitation of the modelling methodology one finds in practical applications is the fact that, often, limited funding for the transportation study results in a partial modelling, where important aspects of the methodology (as public transport modal choices) are grossly simplified or neglected altogether. The model users obtain of course what they are paying for, but a too restrictive financial appraisal of the modelling phase might, and sometimes does, lead to substantial qualitative errors in the conclusions! This type of error is also considered by [43].

6 Conclusion

We have developed a general outline of the context, basic tools and methods in transportation modelling, from which links with operations research have been pointed out, as well as with the subject of other lectures of the workshop.

It is hoped that the presentation has made clear the great diversity of research areas in transportation which can and should benefit from the developments and algorithms of continuous and discrete optimization, as well as other branches of operations research. The joint adventure of the two fields has already lasted for the best of forty years, and the multiplicity and importance of the remaining questions is such that one can safely anticipate a much longer fruitful cooperation.

References

[1] R+D in advanced road transport telematics in Europe (DRIVE90). Technical Report DRI-201, DG XIII, EEC, Brussels, March 1990.

[2] M. Acutt and J. Dodgson. The impact of economic policy instruments on greenhouse gas emission in australian transport. In D. Henscher, J. King, and T. Oum, editors, *Transport Policy, volume 3 of the Proceedings of the 7th World Conference on Transport Research*, pages 321–334, Kidlington, UK, 1996. Pergamon.

[3] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin. *Network flows, Theory, Algorithms, and Applications.* Prentice-Hall, Englewood Cliffs, New Jersey, USA, 1993.

[4] P. Stopher amd M. Lee-Gosselin. *Understanding travel behaviour in an era of change.* Pergamon, Kidlington, UK, 1996.

[5] M. J. Beckman, C. B. McGuire, and C. B. Winston. *Studies in the economics of transportation.* Yale University Press, New Haven (USA), 1956.

[6] M. E. Ben-Akiva, A. de Palma, and I. Kaysi. Dynamic network models and driver information systems. *Transportation Research A*, 25(5):251–266, 1992.

[7] M. E. Ben-Akiva and S. R. Lerman. *Discrete Choice Analysis: Theory and Application to Travel Demand.* MIT Press, Cambridge, USA, 1985.

[8] L. Bianco and P. Toth, editors. *Advanced Methods in Transportation Analysis.* Springer Verlag, Heidelberg, Berlin, New York, 1996.

[9] M. Bierlaire. Evaluation de la demande en trafic : quelques méthodes de distribution. *Annales de la Société Scientifique de Bruxelles*, 105(1-2):17–66, 1991.

[10] M. Bierlaire. A robust algorithm for the simultaneous estimation of hierarchical logit models. GRT Report 95/3, Department of Mathematics, FUNDP, Namur, Belgium, 1995.

[11] M. Bierlaire, T. Lotan, and Ph. L. Toint. On the overspecification of multinomial and nested logit models due to alternative specific constants. *Transportation Science*, (to appear), 1997.

[12] M. Bierlaire and Ph. L. Toint. MEUSE: an origin-destination estimator that exploits structure. *Transportation Research B*, 29(1):47–60, 1995.

[13] D. Braess. Uber ein paradoxon der verkhersplannung. *Unternehmenforschung*, 12:256–268, 1968.

[14] D. Burton and Ph. L. Toint. On an instance of the inverse shortest path problem. *Mathematical Programming, Series A*, 53(1):45–62, 1992.

[15] D. Burton and Ph. L. Toint. On the use of an inverse shortest paths algorithm for recovering linearly correlated costs. *Mathematical Programming, Series A*, 63(1):1–22, 1994.

[16] E. Cascetta. Estimation of trip matrices from traffic counts and survey data: a generalised least squares approach estimator. *Transportation Research B*, 18(4/5):289–299, 1984.

[17] L. Clement, D. Peyrton, and M. Frenois. Review of existing land-suse transport models. Technical Report 58, CERTU, Lyon, France, 1996.

[18] M. Cremer and H. Keller. A new class of dynamic methods for the identification of origin-destination flows. *Transportation Research B*, 21(2):117–132, 1987.

[19] S. Dafermos. Traffic equilibrium and variational inequalities. *Transportation Science*, 14:42–54, 1980.

[20] S. Dafermos. The general multimodal network equilibrium problem with elastic demand. *Networks*, 12:57–72, 1982.

[21] S. Dafermos and A. Nagurney. On some traffic equilibrium theory paradoxes. *Transportation Research B*, 18:101–110, 1984.

[22] A. de Palma, P. Hansen, and M. Labbé. Commuters' paths with penalties for early or late arrival. *Transportation Science*, 24(4), 1990.

[23] R. B. Dial. A probabilistic multipath traffic assignment algorithm which obviates path enumeration. *Transportation Research*, 5(2):83–111, 1971.

[24] R. M. Downs and D. Stea. *Maps in minds.* Harper and Row, New York, 1977.

[25] Y. Ermoliev and R. J.-B. Wets, editors. *Numerical Techniques for Stochastic Programming.* Springer Verlag, Heidelberg, Berlin, New York, 1988.

[26] M. Florian, editor. *Traffic equilibrium methods.* Springer Verlag, New York, 1976. Lecture Notes in economics and mathematical systems 118.

[27] M. Florian. Mathematical programming applications in national, regional and urban planning. In M. Iri and K. Tanabe, editors, *Mathematical Programming: recent developments and applications*, pages 57–82, Dordrecht, NL, 1989. Kluwer Academic Publishers.

[28] R. L. Francis, L. F. McGinnis, and J. A. White. *Facility Layout and Location.* Prentice-Hall, Englewood Cliffs, New Jersey, USA, 1992.

[29] M. Frank. The Braess paradox. *Mathematical Programming*, 20:283–302, 1981.

[30] N. H. Gartner. Optimal traffic assignment with elastic demands: a review. *Transportation Science*, 14:192–208, 1980.

[31] M. Germain, Ph. Toint, and H. Tulkens. International negotiations on acid rains in Northern Europe: a discrete time iterative process. In A. Xepapadeas, editor, *Economic policy for the Environment and natural resources: techniques for the management and control of pollution*, pages 217–236, London, 1996. Edward Elgar Publishing.

[32] M. Germain, Ph. Toint, and H. Tulkens. Financial transfer to ensure cooperative international optimality in stock pollutant abatement. Technical Report 97/4, Department of Mathematics, FUNDP, Namur, Belgium, 1997.

[33] J. D. Griffiths, editor. *Mathematics in Transport Planning and Control.* clarendon Press, Oxford, UK, 1992.

[34] Å. Hallefjord, K. Jørnsten, and S. Storøy. Traffic equilibrium paradoxes when travel demand is elastic. Working paper, University of Bergen, Bergen, Norway, 1990.

[35] R. Hammerslag. Dynamic assignment in the three dimensional timespace: mathematical specification and algorithm. US-Italy Joint Seminar on Urban Traffic Networks, June 1989.

[36] D. Hensher, J. King, and T. Oum, editors. *Travel Behaviour.* Pergamon, Kidlington, UK, 1996. volume 1 of the Proceedings of the 7th World Congress on Transport Research.

[37] P. Jones, editor. *Developments in Dynamic and Activity-Based Approaches to Travel Analysis*, Aldershot, UK, 1990. Avebury.

[38] H. Julien and O. Morellet. What kind of relationship between model choice and forecast traffic for long distance trips? 5th International Conference on Travel Behaviour, Aix-en-Provence, 1987.

[39] P. Kall and S. W. Wallace. *Stochastic Programming.* J. Wiley and Sons, Chichester, England, 1994.

[40] H. Keller and G. Ploss. Real-time identification of O-D network flows from counts for urban traffic control. In N. H. Gartner and N. H. M. Wilson, editors, *Transportation and traffic theory*, pages 267–284, New-York, 1987. Elsevier. Proceedings of the Tenth International Symposium on Transportation and Traffic Theory, July 8–10 1987, MIT, Cambridge, Massachusetts.

[41] A. Khasnabis and B. Chaudry. Transportation-land us interaction: the US experience and future outlook. In D. Henscher, J. King, and T. Oum, editors, *Transport Policy, volume 3 of the Proceedings of the 7th World Conference on Transport Research*, pages 21–30, Kidlington, UK, 1996. Pergamon.

[42] F. Leurent. Cost versus time equilibrium over a network. *Transportation Research Record*, 1443:84–91, 1994.

[43] F. Leurent. *Pour certifier un modèle.* PhD thesis, Ecole National des Ponts et Chausées, Paris, 1996.

[44] M. J. Lighthill and G. B. Whitnam. On kinematic waves ii: a theory of traffic flow on long crowded roads. *Proceedings of the Royal Society, London*, 229A, 1955.

[45] R. F. Love, J. G. Morris, and G. O. Wesolowsky. *Facilities location. Models & methods.* Elsevier (North Holland), New York, 1988.

[46] J. L. Madre and J. Maffre. Toujours plus loin ... mais en voiture. *INSEE Première*, 417, 1995.

[47] McNally and Recker. *Activity-Based Models of Travel Behaviour: Evolution and Implementation.* Pergamon, Kidlington, UK, 1996.

[48] D. K. Merchant and G. L. Nemhauser. A model and an algorithm for the dynamic traffic assignment problems. *Transportation Science*, 12:183–199, 1978.

[49] D. K. Merchant and G. L. Nemhauser. Optimality conditions for a dynamic traffic assignment model. *Transportation Science*, 12:200–209, 1978.

[50] A. Nagurney. Computational comparisons of algorithms for general asymmetric traffic equilibrium problems with fixed and elastic demand. *Transportation Research B*, 20(1):78–83, 1986.

[51] A. Nagurney. An equilibration scheme for the traffic assignment problem with elastic demand. *Transportation Research B*, 22(1):73–79, 1989.

[52] G. L. Nemhauser, A. H. G. Rinnoy Kan, and M. J. Todd. *Optimization*, volume 1 of *Handbooks in Operations Research and Management Science*. North-Holland, Amsterdam, 1981.

[53] M. Papageorgiou, editor. *Concise Encyclopedia of Traffic and Transportation Systems*, Oxford, UK, 1991. Pergamon Press.

[54] M. Patriksson. *The Traffic Assignment Problem, Models and Methods*. VSP, Utrecht, NL, 1994.

[55] S. H. Putman. *Integrated urban models*, volume 1. Pion limited, 207 Brondesbury Park, London NW2 5JN, 1983.

[56] S. H. Putman. *Integrated urban models*, volume 2. Pion limited, 207 Brondesbury Park, London NW2 5JN, 1991.

[57] B. Ran and D. Boyce. *Modelling Dynamic Transportation Networks*. Springer Verlag, Heidelberg, Berlin, New York, 1996. second edition.

[58] K. Safwat and T. Magnanti. A combined trip generation, trip distribution, modal split and trip assignment model. *Transportation Science*, 22:14–30, 1988.

[59] A. Sen and T. E. Smith. *Gravity Models of Spatial Interaction Behaviour*. Springer Verlag, Heidelberg, Berlin, New York, 1995.

[60] Y. Sheffi. *Urban Transportation Networks*. Prentice-Hall, Englewood Cliffs, USA, 1985.

[61] R. E. Tarjan. *Data Structures and Network Algorithms*. SIAM, Philadelphia, USA, 1983. CBMS-NSF, Regional Conference Series in Applied Mathematics.

[62] H. Timmermans. *Activity Based Approaches to Transportation Modelling*. Pergamon, Kidlington, UK, 1996.

[63] Ph. L. Toint and L. Wynter. Asymmetric multiclass assignment: a coherent formulation. In J. B. Lesort, editor, *Transportation and Traffic Theory*, pages 237–260, Oxford, U.K., 1996. Pergamon.

[64] J. Wardrop. Some theoretical aspects of road traffic research. *Proceedings of the Institute of Civil Engineers, part II*, 1:325–378, 1952.

[65] F. V. Webster, P. H. Bly, and N. J. Paulley, editors. *Urban Land-use and Transport Interaction, Policies and Models*. Avebury, Aldershot, UK, 1988.

[66] B. W. Wie, T. L. Friesz, and R. L. Tobin. Dynamic user optimal traffic assignment: a control theoretic formulation. presented at the US-Italy Joint Seminar on Urban Traffic Networks, Capri, 1989.

[67] B. W. Wie, T. L. Friesz, and R. L. Tobin. Dynamic user optimal traffic assignment on congested multidestination networks. *Transportation Research B*, 24(6):431–442, 1990.

[68] A. G. Wilson. *Entropy in urban and regional modelling.* Pion, London, 1970.

[69] S. Yagar. Dynamic traffic assignment by individual path minimization and queing. *Transportation Research*, 5:179–196, 1970.

[70] S. Yagar. Emulation of dynamic equilibrium in traffic networks. In M. Florian, editor, *Traffic equilibrium methods*, New York, 1976. Springer Verlag. Lecture Notes in economics and mathematical systems 118.