An iterative process for international negociations on acid rain in Northern Europe using a general convex formulation

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An iterative process for international negotiations on acid rain in Northern Europe using a general convex formulation

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Abstract

This paper proposes a dynamic game theoretical approach of international negotiations on transboundary pollution. This approach is distinguished by a discrete time formulation and by a suitable formulation of the local information assumption on cost and damage functions: at each stage of the negotiation, the parties assign the best possible cooperative state, given the available information, as an objective for the next stage. It is shown that the resulting sequences of states converges from a non-cooperative situation to a Pareto optimum in a finite number of stages. Furthermore, a financial transfer structure is also presented, which guarantees that the desired sequence of states is individually rational and strategically stable if one starts from a Nash equilibrium. The concepts are applied in a numerical simulation of the SO_2 transboundary pollution problem related to acid rain in Northern Europe. This simulation shows the need for an improved formulation of the financial transfers if one starts from another initial state. Such a formula is proposed and tested numerically.

1 Introduction

Modelling international negotiations on transboundary pollution has enjoyed a lot of attention in the recent past. Amongst the many contributions in this area, Chander and Tulkens (1995) and Chander and Tulkens (1996) present a general dynamic framework for such models, which is applied by Kaitala *et al.* (1995) and Germain *et al.* (1996) in the context of the "acid rain game" in Northern Europe, that is the problem of negotiating coordinated reductions in SO_2 emission levels in Finland, Russia and Estonia. The framework proposed by these authors does not assume that the information available to the negotiating parties is *global*, as is for instance requested by Mäler (1989) or Kaitala and Pohjola (1994), but rather supposes that this information is *local* in that it is limited to values in the neighbourhood of the current emission abatement costs and pollution damage costs to the environment. Germain *et al.* (1996) assume that negotiations are separated into well determined and distinct stages, and that the negotiating parties know, at each stage, their emission abatement and damage cost function only up to certain thresholds determined by exogenous technological constraints[1]. At each stage, the parties then assign as a

[1] Following Tulkens (1979), whose model is inspired by the literature on "resource allocation processes" (Arrow and Hurwicz (1977), Malinvaud (1970) and Drèze and de la Vallée Poussin (1971)), Kaitala *et al.* (1995) assume

common objective the best cooperative state they can determine, given the available information. As these thresholds evolve with the progress made in reducing pollution, they show that the Pareto optimum is reached in a finite number of such negotiation stages. Besides indicating that only a finite number of negotiation stages is necessary to reach the optimum, their approach has the advantage of clarifying the associated time scale, where negotiations and periods between them are clearly distinguished.

The same authors also succeed in adapting to a dynamic context a financial transfer scheme originally proposed by Chander and Tulkens (1996) in a static context, which guarantees that it is of the interest of all negotiating parties individually to cooperate at each stage of the process. This transfer scheme additionally results in a distribution of the depollution costs which is strategically stable, in that no partial coalition of the parties can provide lower costs to its members than is obtained by full cooperation.

The theoretical results of Germain *et al.* (1996) assume that the procedure starts from a Nash equilibrium and that the environmental damage functions are linear for each of the involved regions. The first contribution of this paper is to consider possibly strictly convex damage functions. Because of the difficulty inherent in estimating damage functions, it is indeed highly desirable to allow more than a single representation, in order to analyze how crucial variables (like emissions, gains resulting from cooperation and transfers) are affected by the chosen representation. The negotiation procedure and its application to the acid rain problem in Northern Europe are presented in Section 1.

On the other hand, the choice of strictly convex damage functions has two main consequences. First, and at variance with the linear case, we have to modify the equations for the financial transfers in order to apply them to our framework[2]. Moreover, since strictly convex damage functions no longer guarantee monotonically decreasing emission levels, the reasoning of Chander and Tulkens (1996) or Germain *et al.* (1996) leading to coalitional rationality is no longer applicable. This modification is discussed in Section 2.

The second contribution of the paper is to test individual and coalitional rationality when the initial state is not a Nash equilibrium. This type of initial situation is interesting in itself, but also because of the difficulty of verifying that a given initial situation is indeed a Nash equilibrium. A first complication, compared to the previously cited contributions, is that the Nash equilibrium

in a similar manner that the regions only know their cost functions at the margin. They are then able to define a pollution reduction program such that emission levels converge to their optimal values E_i^*, thus realizing the Pareto optimum.

Although a clear improvement on assuming a global knowledge of the cost functions, this procedure remains problematic because, being defined in continuous time, it implies that an interregional negotiation occurs at each moment in time. This is of course impractical. Moreover, this approach has the further drawback that its time scale is not well defined since periods between two successive negotiations are not modelled. Kaitala *et al.* thus propose to modify the dynamic of the process by adjusting a certain speed parameter to suit any specific time scale definition. This technique is interesting but does not resolve the difficulty that the time scale itself could be better defined.

[2] This is notably different from the model introduced by Kaitala *et al.* (1995), where continuous time allow direct application of Chander and Tulkens's formula, irrespective of the linear or strictly convex nature of the damage functions.

which plays a role in the calculation of the financial transfers (and therefore in the individual and coalitional rationality properties) is now constrained by the limited nature of the information on the cost functions available at the starting situation. Moreover, even if the transfer formula is adapted to handle this problem, it appears that it does not guarantee any form of rationality. As a consequence, we propose, in Section 3, a new transfer formulation which no longer depends on the Nash equilibrium, but rather on the initial state. We also test the validity of this new formulation on the Northern European acid rain problem in the same section.

2 An iterative process for international negotiations

2.1 Problem description

The problem may be described as follows. There are n regions in which some atmospheric pollutant (such as SO_2) is emitted at each period. Let the vector $E_t = (E_{1,t}, \ldots, E_{n,t})^T$ describe these non-negative emission levels at time t ($t = 0, 1, \ldots$). Once emitted, this pollutant is transported by winds between the regions, this interregional transport being described by the $n \times n$ matrix A, whose i, j-th element $a_{ij} \geq 0$ is the fraction of the pollutant emitted in region i and deposited in region j. If B is the n-vector of pollutant depositions with natural origin, the vector $Q_t = (Q_{1,t}, \ldots, Q_{n,t})^T$ of total pollutant depositions in the n regions at time t satisfies the relation

$$Q_t = Q(E_t) = AE_t + B, \tag{2.1}$$

where A and B are supposed to be constant and exogenous. We assume that (2.1) holds at each moment in time, so that the fact that B is constant then implies that the pollutant does not accumulate in the regions.

Two cost functions are associated with each region. The first, denoted $C_i(E_{i,t})$, expresses the emission abatement costs required by an emission level $E_{i,t}$ in region i during period t. We assume that $C_i(\cdot)$ is decreasing for all positive emission levels E_i, which simply means that every emission reduction generates costs for the emitting region. The second cost function, denoted $D_i(Q_{i,t})$, measures the monetary value of the damage to the environment caused by a deposition Q_i in region i during period t. $C_i(\cdot)$ and $D_i(\cdot)$ are assumed to be continuous and convex. As a result, we may associate, with each region, an aggregated cost function

$$J_i(E_t) = C_i(E_{i,t}) + D_i(Q_i(E_t)), \tag{2.2}$$

where the depositions $Q_i(E_t)$ are given by (2.1). Equation (2.2) clearly states that the environmental damages in region i depend on the emissions of the other regions. If the countries act optimally, i.e. if they *minimize the sum of all their costs*, the vector of emission levels is obtained as solution of the program

$$\min_{E_t} J(E_t) \tag{2.3}$$

where

$$J(E_t) \stackrel{\text{def}}{=} \sum_{i=1}^{n} J_i(E) \tag{2.4}$$

and such that (2.1) holds and

$$E_{i,t} \geq 0 \quad (i = 1, \ldots, n). \tag{2.5}$$

Let $E^* = (E_1^*, \ldots, E_n^*)$ be the vector of emissions at the Pareto optimum. Because A and B are constant and because the functions C_i and D_i does not explicitly depend on time, E^* does not depend on time either. Moreover, this optimum is unique because of the convexity of the $C_i(\cdot)$ and $D_i(\cdot)$.

Starting from a sub-optimal situation (described below) described by the pair (E_0, Q_0), we assume that the regions are involved in a negotiation process whose purpose is to bring them together to an optimal state where the sum of all costs is minimum. In their description of the negotiation process, some authors (see Kaitala and Pohjola, 1994, for instance) assume that the parties know the costs functions for all possible values of their arguments from the beginning, in which case the optimal state (E^*, Q^*) may be calculated and reached in a single stage. This is called a "one shot" game. The difficulty is that this assumption may be unrealistic when the current emission levels and depositions are very different from the optimal ones: such an extensive knowldege of the cost functions may be very hard to obtain, or may be too unreliable to be used in effective interregional negotiations.

In this context, it seems more reasonable to assume that the negotiations consist of several successives stages $t = 0, 1, \ldots$, and that, at each such stage, *the regions only know their cost functions in a finite neighbourhood of the current emission levels and depositions*, that is above a certain threshold strictly below these levels. Specifically, each region is assumed to know, for each period t, the values of the emission abatement cost $C_i(E_i)$ for all values of the emission levels

$$F_{i,t} \leq E_i, \tag{2.6}$$

where the lower bounds $F_{i,t}$ satisfy the conditions $0 \leq F_{i,t} < E_{i,t}$ for each i and each t. Similarly, each region is assumed to know its damage function $D_i(Q_i(E))$ for each value of the vector of emissions $(E_1, \ldots, E_n)^T$ above the same thresholds.

At the t-th negotiation stage, the parties agree on a program to bring their emission levels from E_t to the values E_{t+1} corresponding to a solution of the local optimization problem (2.3) in the domain defined by the constraints (2.1) and (2.6)[3]. Once these emissions levels are attained, the bounds are updated to reflect the new information obtained during the period, according to

$$F_{i,t+1} = \min[F_{i,t}, \tau_{i,t+1} E_{i,t+1}] \tag{2.7}$$

for $i = 1, \ldots, n$, where

$$0 \leq \tau_{i,t+1} \leq \tau \tag{2.8}$$

for some constant $0 \leq \tau < 1$. The procedure may then be repeated for the $(t+1)$-th negotiation stage. Of course, it may well happen that $E_{t+1} = E_t$ for some t, in which case no further action is requested from the parties and the procedure is then terminated. *Note that (2.7) ensures that the*

[3]Note that $J(\cdot)$ is the sum of convex function and is therefore convex. Hence the local problem (2.3)+(2.1)+(2.6) is also convex.

domains defined by the constraints (2.6) for successive t are nested, and hence that termination occurs at stage t if $E_{i,t} > F_{i,t}$ for all i.

One then has the following important result, which states that the emission levels (and hence depositions) converge to a state corresponding to a Pareto optimum, provided the level set

$$\mathcal{L}_0 \stackrel{\text{def}}{=} \{E \mid J(E) \leq J(E_0) \text{ and } E_i \geq 0 \quad (i = 1, \ldots, n)\} \tag{2.9}$$

is bounded. This assumption is fairly weak. It is for instance satisfied if the emission levels are not only non-negative but also bounded above (a very plausible situation), or if

$$\lim_{||E|| \to \infty} \sum_{i=1}^{n} D_i(Q_i(E)) = \infty. \tag{2.10}$$

This latter condition is also natural in our context, since it only reflects the fact that increasing pollutant emission cause increased damage, maybe not locally, but somewhere within the regions under consideration.

Theorem 1 *Under the conditions stated above, the sequence $\{E_t\}$ generated by the procedure starting from any initial situation E_0 converges to a solution of the problem (2.3)+(2.1)+(2.5). Furthermore, this convergence occurs in a finite number of stages if all limiting values of the emissions are strictly positive.*

The interested reader is referred to Germain *et al.* (1996) for a proof.

2.2 An application to Northern European negotiations on acid rain

We now examine the impact of the model and theory developed in the previous section on the "acid rain game" in Northern Europe. Specifically, the problem is for Finland (divided in three regions), three regions of Russia and Estonia to agree on a program for reducing SO_2 emissions, which should limit acid rain and its associated damages. One of the main goals of the paper is to discuss coalitional rationality of the negotiation procedure for convex cost functions, in particular in cases where the assumptions of Chander and Tulkens (1995) and Chander and Tulkens (1996) do not hold. Thus, we assume here that $C_i(\cdot)$ and $D_i(\cdot)$ are both quadratic. More precisely, the emission abatement costs are assumed to be, in the interval $[0, \tilde{E}_i]$, of the form

$$C_i(E_i) = \gamma_i + \alpha_i(\tilde{E}_i - E_i) + \beta_i(\tilde{E}_i - E_i)^2 \tag{2.11}$$

for each $i = 1, \ldots, n$ while the damage functions may be written as

$$D_i(Q_i) = \frac{1}{2}\pi_i Q_i^2 \tag{2.12}$$

for each i. Both these costs are expressed in millions of 1987 Finnish Marks (MFIM) and the depositions are expressed in thousands of tons per year (KT/Y). The parameters α_i, β_i, γ_i and $\tilde{E}_i$ are known and positive. Their values, together with those of B and the initial levels E_0, Q_0 in 1987, are given in Table 6 of the Appendix. The parameters π_i are estimated by applying the revealed preference method of Kaitala and Pohjola (1994), which relies on the assumption that the initial state corresponds to a Nash equilibrium between the seven regions[4]. The transport coefficients (the matrix A) is given in Table 7 of the Appendix. The information thresholds for the costs and damage functions are determined using the formula (2.7) where $\tau_{i,t} = 0.95$ for all i and t, which means that these functions are known for all emission levels above 95% of the current ones.

Table 1 and Figure 1 show the evolution of the SO_2 emission levels that result from the procedure described in the previous section. In the figure (and all following ones), the regions are identified as follows: Northern Finland's trajectory is represented by a dashed line, Central Finland's by "x", Southern Finland's by "+", Kola by stars, Karelia's by a continous line, St Petersburg by circles and Estonia by a dotted line. The final state ($t = 28$) is the Pareto optimum for the problem and corresponds to that reported by Kaitala *et al.* (1995). The numerical computations were performed by using the LANCELOT package for nonlinear optimization by Conn *et al.* (1992).

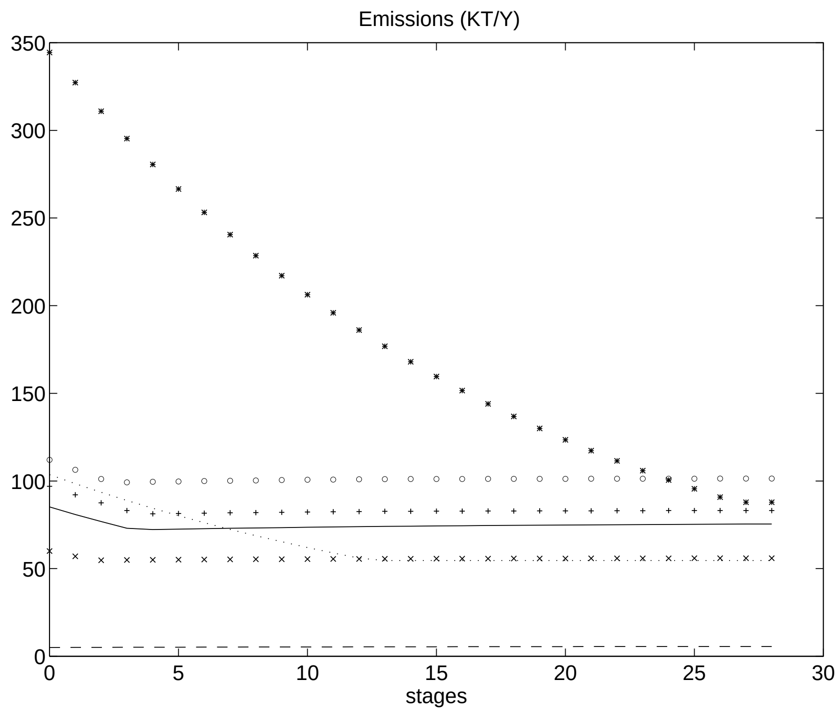

Figure 1: Evolution of the SO_2 emissions

[4]This equilibrium is formally defined in paragraph 3.1. If the initial state is such an equilibrium, it is characterized by the n conditions

$$C_i'(E_{i,0}) = \alpha_i + 2\beta_i(\bar{E}_i - E_{i,0}) = \pi_i Q_{i,0} = D_i'(Q_{i,0}), \quad (i = 1, \ldots, n).$$

The constants α_i, β_i, $\bar{E}_i$, $E_{i,0}$ and $Q_{i,0}$ being observable (see Table 6), it is then possible to compute the π_i.

t	NF	CF	SF	Ko	Ka	SP	Es
0	$\boxed{5.0000}$	60.0000	97.0000	350.0000	85.0000	112.0000	104.0000
1	5.0007	60.0378	96.9866	344.4493	85.1660	112.0175	103.6623
2	5.0493	57.0359	92.1373	327.2269	80.9077	106.4167	98.4792
3	5.0950	$\boxed{54.7550}$	87.5305	310.8655	76.8623	101.0958	93.5553
4	5.1363	54.9448	83.1539	295.3223	73.0192	$\boxed{99.2097}$	88.8775
5	5.1726	55.0533	$\boxed{81.2824}$	280.5561	$\boxed{72.2965}$	99.5378	84.4336
6	5.2056	55.1258	81.4751	266.5283	72.5423	99.7535	80.2119
7	5.2369	55.1946	81.6582	253.2019	72.7758	99.9585	76.2013
8	5.2666	55.2600	81.8321	240.5418	72.9977	100.1532	72.3913
9	5.2949	55.3221	81.9974	228.5147	73.2084	100.3381	68.7717
10	5.3217	55.3812	82.1544	217.0890	73.4086	100.5139	65.3331
11	5.3472	55.4372	82.3035	206.2345	73.5988	100.6808	62.0665
12	5.3714	55.4905	82.4452	195.9228	73.7795	100.8394	58.9631
13	5.3944	55.5411	82.5798	186.1267	73.9511	100.9900	56.0150
14	5.4163	55.5838	82.6700	176.8203	74.1015	101.0775	$\boxed{54.5753}$
15	5.4371	55.6191	82.7178	167.9793	74.2317	101.1045	54.5776
16	5.4569	55.6526	82.7632	159.5804	74.3554	101.1302	54.5797
17	5.4757	55.6844	82.8064	151.6013	74.4729	101.1547	54.5817
18	5.4936	55.7146	82.8474	144.0213	74.5845	101.1778	54.5837
19	5.5105	55.7434	82.8864	136.8202	74.6906	101.1999	54.5855
20	5.5266	55.7706	82.9234	129.9792	74.7913	101.2208	54.5873
21	5.5419	55.7966	82.9585	123.4802	74.8870	101.2407	54.5889
22	5.5565	55.8212	82.9919	117.3062	74.9779	101.2596	54.5905
23	5.5703	55.8446	83.0237	111.4409	75.0643	101.2775	54.5920
24	5.5834	55.8668	83.0538	105.8689	75.1463	101.2946	54.5934
25	5.5959	55.8879	83.0824	100.5754	75.2243	101.3108	54.5948
26	5.6077	55.9080	83.1096	95.5467	75.2983	101.3262	54.5960
27	5.6190	55.9270	83.1355	90.7693	75.3687	101.3408	54.5973
28	5.6259	55.9387	83.1513	$\boxed{87.8371}$	75.4118	101.3497	54.5980

Table 1: Evolution of the SO_2 emissions

As shown by Table 1, 28 negotiation stages are necessary, given our assumptions, to reach the optimum. This number of stages could of course be reduced if the values chosen for $\tau_{i,t}$ were smaller. If we now compare the initial and final states, we note that the reductions vary considerably between regions. While the emissions of Central and Southern Finland, Karelia and St Petersburg are slightly reduced, the reduction is much more substantial for Kola and Estonia. We even note the remarkable *increase* for Northern Finland. A similar situation occured in Kaitala *et al.* (1995), where it is explained as follows. Observe that, at the Nash equilibrium, the marginal emission abatement costs C_i' in the various regions are quite different from each other: some may be very high, as is precisely the case with Northern Finland. Observe also that the sum of marginal damages $\sum_{j=1}^{n} a_{ji}\pi_j Q_j$ being now an increasing function of the depositions, decreases with the reduction of the latter. Therefore, at some stage t and for some region i, the sum of marginal damages becomes equal to its marginal emission abatement cost (implying $\Delta E_i = 0$ for this i), while such equality does not (yet) hold for the other regions $j \neq i$. The reduction in depositions *achieved by these other regions* then induce a decrease of $\sum_{j=1}^{n} a_{ji}\pi_j Q_j$ while C_i' does not change, thus inducing a sign reversal in ΔE_i.

Table 1 also shows that, with the exception of Kola, none of the regions which, in fine, reduce their emission levels, do so monotonically. The boxed entries in Table 1 indicate, for each region, the minimum emission reductions achieved in the process. Northern Finland is at its minimum at the initial state, Central Finland achieves it at the third stage, St Petersburg at the fourth, Southern Finland and Karelia at the fifth, Estonia at the fourteenth and Kola at the final stage only. That some regions do increase their emissions after this minimum has been reached may again be explained as above, noting that the steady decrease in Kola's emissions induces a decrease in depositions in the whole area, implying in turn a decrease in marginal damage and hence raising the depollution opportunity in several regions.

If one compares the optimal emission obtained here with their corresponding levels in the case where the damages are linear (see Germain *et al.*, 1996) and the assumption on the initial coalition different, one notes that the orders of magnitude and the number of stages (for the same value of τ) remain similar, despite the different assumptions on the initial state. As one of our aims was to verify the impact of alternative formulations for the damage functions, this observation is reassuring. However, one can nevertheless see two differences with respect to the linear case. The first is that emissions are monotonically decreasing in the linear setting, which is not the case here. This is because marginal damages are constant for linear damage functions, and are therefore not influenced by the repeated emission reductions for Kola. The second is that the optimal emission levels are higher in the quadratic case, even if their order of magnitude remains comparable. Indeed, the marginal damages (and thus the incentive for further depollution) decrease with the emission levels and pollutant depositions when the damage functions are quadratic.

Let us now consider the total costs incurred by each region in the transition from the initial Nash equilibrium to the final Pareto optimum. More precisely, we report in Table 2 the values

$$J_i(E^*) - J_i(E_0) = C_i(E^*) - C_i(E_0) + D_i(E^*) - D_i(E_0)$$

for $i = 1, \ldots, n$, where E^* is the optimal emission level reached at stage 28. The table also reports the same costs aggregated for the whole of Finland, the whole of the three regions of Russia and Estonia. Figure 2 presents the evolution of these costs over the 28 negotiations stages.

Finland			Russia			Estonia
NF	CF	SF	Ko	Ka	SP	Es
-526.92	-68.85	-41.87	80.08	-69.94	-82.62	9.88
-637.63			-72.47			9.88

Table 2: Total costs $(J_i(E^*) - J_i(E_0))$ per region and country (MFIM)

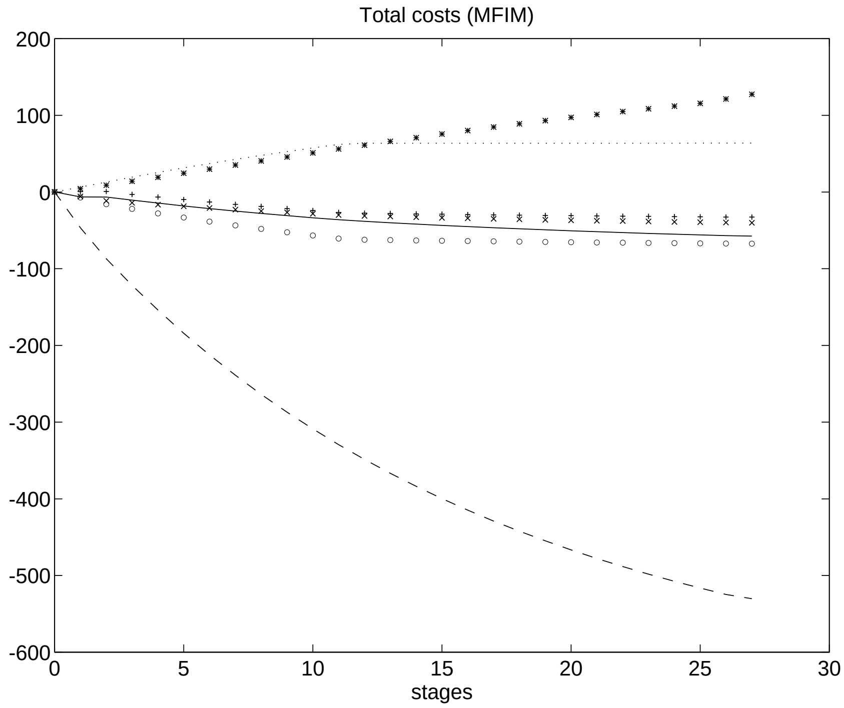

Figure 2: Evolution of the total costs per region

These results show that the emission reduction program does considerably profit to Northern Finland and, in a lesser measure, to Central and Southern Finland, Karelia and St Petersburg, but costs to Kola and Estonia. The cooperation of these regions in an international program is thus unlikely. Also, nothing guarantees that some coalitions (subsets) of regions may not be capable to induce costs for their members that are lower than the costs at the optimum. One possible way to discourage such coalitions is to provide financial transfers between the regions, that would compensate for the incurred costs. The definition of such a strategy is the object of the next section.

3 Coalitional rationality starting from a Nash equilibrium

3.1 Definition of the financial transfers

The aggregate reduction of total costs $J(E^N) - J(E^*)$ (also called the "ecological surplus") is the sum of the gains and of the losses of the different regions. The gains of the winning regions are necessarily at least equal (in absolute value) to the losses of the loosing regions. Those gains are thus more than sufficient to offer to Kola and Estonia a financial transfer that would reduce their costs below the level supported initially, while maintaining considerable advantages for the other regions.

However, the surplus of the winning regions are only known at the end of the reduction program, once the emission levels E^* are achieved. Thus they cannot be used in any practical negotiation process between the parties during the program. Instead, one needs a key to share whatever benefit are achieved at a given negotiation stage between the regions, in order to ensure that each of them continues to cooperate within the program. The aim of this section is to show that such financial transfers can indeed be found.

We start by characterizing the non-cooperative Nash equilibrium between the regions when information is limited. At this equilibrium, each region is assumed to choose its emission level such that the sum of its depollution and damage costs are minimized, given the emissions of the other regions and the information available on the cost functions. Formally, the equilibrium is characterized, at period t, by the emission vector $E_t^N = (E_{1,t}^N, \ldots, E_{n,t}^N)$ which is the simultaneous solution of n optimization problems given by

$$\min_{E_{i,t}} J_i(E_{i,t}) = C_i(E_{i,t}) + D_i(Q_{i,t}(E_{i,t})) \quad (i = 1, \ldots, n) \tag{3.1}$$

subject to the constraints (2.1) and (2.6), where, for each i, $E_j = E_{j,t}^N$, $(j \neq i)$ and $F_{i,t}$ are given. The total surplus obtained after negotiation stage t with respect to the Nash equilibrium is the sum of the different regions' gains or losses, that is

$$\Delta J_t = J_t - J_t^N = \sum_{i=1}^{n} \Delta J_{i,t} = \sum_{i=1}^{n} [\Delta C_{i,t} + \Delta D_{i,t}], \tag{3.2}$$

where we have used the convention that Δ prefixing a variable stands for the difference between this variable and that at the Nash equilibrium (3.1) (thus $\Delta C_{i,t} = C_{i,t} - C_{i,t}^N$). Because of the quadratic form of the damage function (2.12), we obtain that

$$\Delta D_{i,t} \leq \sum_{j=1}^{n} \frac{\partial D_i}{\partial E_j}(E_{j,t})[E_{j,t} - E_{j,t}^N] = D_i'(Q_{i,t}) \sum_{j=1}^{n} a_{ij} \Delta E_{j,t}. \tag{3.3}$$

Combining (3.2) and (3.3), we see that

$$\begin{aligned} \Delta J_t &\leq \sum_{i=1}^{n} \Delta C_{i,t} + \sum_{i=1}^{n} D_i'(Q_{i,t}) \sum_{j=1}^{n} a_{ij} \Delta E_{j,t} \\ &= \sum_{j=1}^{n} [\Delta C_{j,t} + \Delta E_{j,t} \sum_{i=1}^{n} a_{ij} D_i'(Q_{i,t})] \stackrel{\text{def}}{=} \sum_{j=1}^{n} \Delta G_{j,t}, \end{aligned} \tag{3.4}$$

defining the $\Delta G_{j,t}$. These quantities can be seen as a linear approximation of the total surplus with respect to the Nash equilibrium obtained when region j decreases its emissions[5].

We have now the following result.

Theorem 2 *At each stage t of the negotiation process and for all regions ($j = 1, \ldots, n$), one has that*

$$\Delta G_{j,t} \leq 0 \ \textit{ and } \ \Delta R_{j,t} \leq 0, \tag{3.5}$$

where $\Delta G_{j,t}$ is defined in (3.4) and

$$\Delta R_{j,t} \stackrel{\text{def}}{=} \Delta D_{j,t} - D_j'(Q_{j,t}) \sum_{i=1}^{n} a_{ji} \Delta E_{i,t}. \tag{3.6}$$

Proof. For stage t, consider the functions

$$G_j(E_j) \stackrel{\text{def}}{=} C_j(E_j) + E_j \sum_{i=1}^{n} a_{ij} D_i'(Q_{i,t}), \quad j = 1, ..., n,$$

where $Q_{i,t}$ are considered as parameters. This definition implies that

$$\Delta G_{j,t} = G_j(E_{j,t}) - G_j(E_j^N) \tag{3.7}$$

because of (3.4), and the convexity of G_j in E_j also yields that

$$G_j(E_{j,t}) - G_j(E_j^N) \leq G_j'(E_{j,t})[E_{j,t} - E_j^N]. \tag{3.8}$$

Now, the derivative of G_j is given by

$$G_j'(E_j) = C_j'(E_j) + \sum_{i=1}^{n} a_{ij} D_i'(Q_{i,t})$$

and the first order optimality conditions of problem (2.3)-(2.1)-(2.6) at stage t imply that

$$G_j'(E_{j,t}) = C_j'(E_{j,t}) + \sum_{i=1}^{n} a_{ij} D_i'(Q_{i,t}) \geq 0 \tag{3.9}$$

when $E_{j,t} \leq E_j^N$ and

$$G_j'(E_{j,t}) = C_j'(E_{j,t}) + \sum_{i=1}^{n} a_{ij} D_i'(Q_{i,t}) = 0 \tag{3.10}$$

otherwise, because the emissions are only bounded from below. Combining (3.7), (3.8), (3.9) and (3.10) then yields the first part of (3.5). The second part of (3.5) immediately follows from (3.3). □

[5] When time is continuous (as in Kaitala *et al.*, 1995) or when the damage functions are linear (as in Germain *et al.*, 1996), inequalities in (3.3) and (3.4) are replaced by equalities, and $\Delta G_{j,t}$ is then exactly the total surplus due to $\Delta E_{j,t}$.

Using this result and the fact that, from (3.6),

$$\begin{aligned}\Delta J_t - \sum_{j=1}^{n} \Delta G_{j,t} &= \sum_{j=1}^{n} \Delta D_{j,t} - \sum_{j=1}^{n} \Delta E_{j,t} \sum_{i=1}^{n} a_{ij} D_i'(Q_{i,t}) \\ &= \sum_{j=1}^{n} [\Delta D_{j,t} - D_j'(Q_{j,t}) \sum_{i=1}^{n} a_{ji} \Delta E_{i,t}] \\ &= \sum_{i=1}^{n} \Delta R_{i,t},\end{aligned} \tag{3.11}$$

we may then specify for each region i and each negotiation stage t, a financial transfer $T_{i,t}$ (negative if received by i or positive if paid) given, for $i = 1, \ldots, n$, by

$$T_{i,t} = -\Delta C_{i,t} - \Delta D_{i,t} + \sum_{j=1}^{n} \delta_{ij,t} \Delta G_{j,t} + \mu_{i,t} \sum_{j=1}^{n} \Delta R_{j,t}, \tag{3.12}$$

where

$$0 \leq \delta_{ij,t} \leq 1, \ \forall i, j, t \ \text{ and } \ \sum_{i=1}^{n} \delta_{ij,t} = 1, \ \forall j, t, \tag{3.13}$$

and

$$0 \leq \mu_{i,t} \leq 1, \ \forall i, t \ \text{ and } \ \sum_{i=1}^{n} \mu_{i,t} = 1, \ \forall t. \tag{3.14}$$

The first two terms of (3.12) are equivalent to levy on region i its gain on total cost, while the third and fourth terms correspond to returning to the region fractions $\delta_{ij,t}$ and $\mu_{i,t}$ of each of the components of the surpluses $\Delta G_{j,t}$ and $\Delta R_{j,t}$, respectively. One immediately verifies that the budget of these transfers is balanced at each period, namely that

$$\sum_{i=1}^{n} T_{i,t} = 0, \ \forall t. \tag{3.15}$$

Moreover, as the total cost for region i with transfers is

$$J_i^T = C_i(E_i) + D_i(E_i) + T_i, \tag{3.16}$$

we have then that

$$\Delta J_i^T = J_{i,t}^T - J_i^N = \Delta C_{i,t} + \Delta D_{i,t} + T_{i,t} = \sum_{j=1}^{n} \delta_{ij,t} \Delta G_{j,t} + \mu_{i,t} \sum_{j=1}^{n} \Delta R_{j,t} \leq 0, \tag{3.17}$$

for $i = 1, ..., n$, because of (3.5). In other words, the total cost of each region is less at the optimum compared with the Nash equilibrium at each negotiation stage. This property, called *individual rationality*, provides a minimum cooperative character to the sequence of negotiations.

Of course, conditions (3.13) and (3.14) do not define the transfers in a univoque manner. All values of the parameters $\delta_{ij,t}$ and $\mu_{i,t}$ satisfying these constraints are adequate. The remaining degrees of freedom may then be used in order to obtain properties of coalitional rationality, as suggested by the theory of cooperative games. Such results have been obtained by Chander and Tulkens (1996) in a context of "global" games and extended by Germain *et al.* (1996) to the case where a discrete time procedure and linear damage functions are considered.

Unfortunately, these results do not extend in full generality (at least from the theoretical point of view) to the case where damage functions are merely strictly convex. In what follows, we outline the reasonning of the cited authors for coalitional rationality and numerically verify that it applies to the framework of acid rains in Northern Europe with quadratic damage costs.

The first step is to describe, at each period t, the emissions of each member of a coalition, as well as those of the regions that are not member. If the coalition $S \subseteq \{1, \ldots, n\}$ is formed at period t, the emission vector $E_t^S = (E_{1,t}^S, \ldots, E_{n,t}^S)$ is determined as follows:

(i) the subvector $\{E_{i,t}^S\}_{i \in S}$ minimizes $\sum_{i \in S} J_i(E_t)$ under the constraints (2.1) and (2.6), where, for $j \notin S$, the values of $E_{j,t} = E_{j,t}^S$ are defined in (ii);

(ii) for all $j \notin S$, $E_{j,t}$ minimized $J_j(E_t)$ under the constraints (2.1) and (2.6), where, for $i \in S$, the values of $E_{i,t} = E_{i,t}^S$ are defined in (i),

the bounds $F_{i,t}$ being given in both cases. One therefore assumes that if a coalition is formed at period t, its members minimize *together* the sum of their total costs for the period, while the other regions react in minimizing their individual total cost for the period. Observe that this situation generalizes both the international Pareto optimum ($S = \{1, \ldots, n\}$) and the Nash equilibrium ($S = \{i\}$ for each i).

Our problem is now to adapt the transfers (3.12) in order to discourage the formation of coalitions[6] under the assumptions stated above. In other words, one should determine, at each period t, a vector of transfers $(T_{1,t}, \ldots, T_{n,t})^T$ such that, for all possible subsets S,

$$\sum_{i \in S} [C_i(E_{i,t}) + D_i(Q_{i,t}) + T_{i,t}] \leq \sum_{i \in S} [C_i(E_{i,t}^S) + D_i(Q_{i,t}^S)], \tag{3.18}$$

where $Q_t^S = AE_t^S + B$. This condition states that no subcoalition S can possibly achieve by itself a cost that is smaller than that it would obtain from full cooperation with transfers.

We now investigate whether, if

$$\delta_{ij,t} = \frac{a_{ij} D_i'(Q_{i,t})}{\sum_{k=1}^n a_{kj} D_k'(Q_{k,t})} \quad \text{and} \quad \mu_{i,t} = \frac{\Delta R_{i,t}}{\sum_{j=1}^n \Delta R_{j,t}}, \tag{3.19}$$

then the transfers given by (3.12) and (3.19) guarantee condition (3.18) at each period, therefore making the international negotiation procedure coalitionaly rational[7].

With linear damage functions, Germain *et al.* (1996) have managed to prove the above results of coalition rationality, ensuring that there is no advantage for a coalition of regions to form and cooperate in a framework more restricted than the full inter-regional cooperation considered here. Following Chander and Tulkens (1996), it could be possible to derive this result for convex damage functions under additional conditions on the marginal damage functions. Unfortunately, these conditions are not verified here. We will therefore verify numerically that the coalitional rationality of the process is obtained with the transfers defined by (3.12) and (3.19).

[6] That is subsets $S \subset \{1, \ldots, n\}$ of regions such that $S \neq \{1, \ldots, n\}$.

[7] It is clear that the parameters defined by (3.19) satisfy (3.13) and (3.14). As a consequence, the transfers are financially balanced in that (3.15) holds.

We finally note that a simpler expression may be obtained for the financial transfers if the choices (3.19) are made. Indeed, substituting (3.19), (3.4) and (3.6) into (3.12) yields that

$$T_{i,t} = -\Delta C_{i,t} + \sum_{j=1}^{n} \delta_{ij,t} \Delta C_{j,t}, \tag{3.20}$$

for $i = 1, \ldots, n$. This latter form may be interpreted as the sum of two partial transfers: the first is *received* by region i to cover its depollution costs, the second being *paid* by region i to finance the reduction of emissions in each region j in a proportion equal to $\delta_{ij,t}$.

3.2 The transfers in the example

The application of formulae (3.12) and (3.19) to the cooperation problem between the regions of Finland, Russia and Estonia leads to the evolution of transfers illustrated by Figure 3. Recall that $T_{i,t} \geq 0$ (≤ 0) means that region i pays (receives) transfers. Estonia and Kola are the principal recipients of transfers, i.e. the regions that reduce most their emissions and whose global costs *without* transfers increase with international cooperation (see Figure 2). The principal payer is Northern Finland, the region that profits most from cooperation. Figure 3 also shows that Southern Finland and St Petersburg receive small transfers during the first stages, but they soon become payers, as Karelia and Northern and Central Finland.

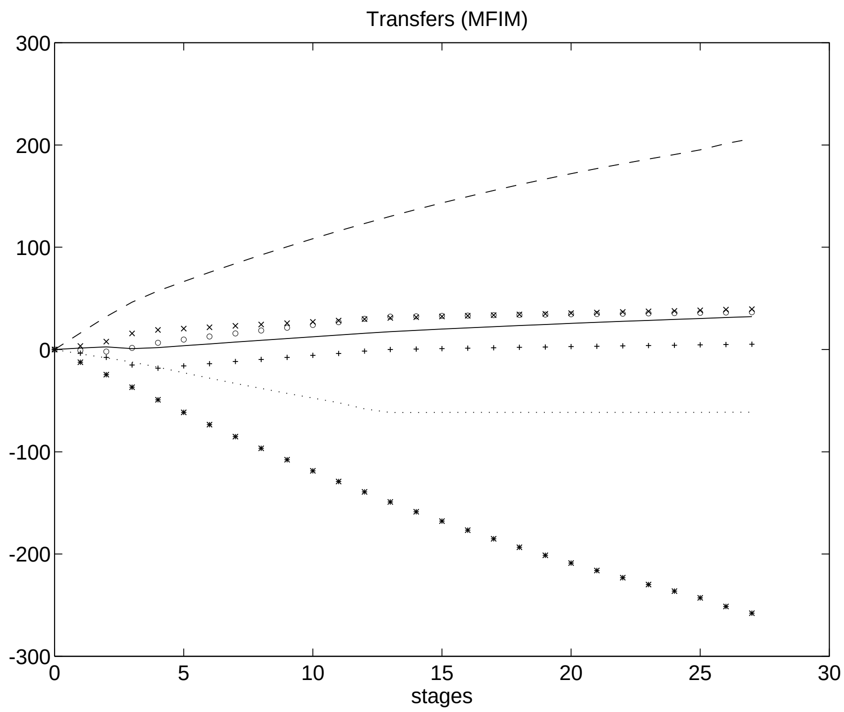

Figure 3: Evolution of the transfers per region

The presence of transfers doesn't imply that the benefits of cooperation are evenly distributed among regions, as shown (in absolute terms) by the third column of Table 3[8]. The fact that the total cost of each region decreases illustrates the property of individual rationality of the negotiation process with transfers. The last column of Table 3 shows that the distribution of the surplus is also uneven in relative terms. Because of its low initial costs (due to its very low marginal damage), Kola obtains, taking the transfers into account, the most important relative reduction of total costs, followed by Northern Finland[9].

	$J_i(E^*) - J_i(E_0)$	$T_i(E^*)$	$J_i^T(E^*) - J_i(E_0)$	$\frac{J_i^T(E^*) - J_i(E_0)}{J_i(E_0)}$
NF	-526.92	209.20	-320.72	-27.84 %
CF	-68.85	39.41	-29.44	-4.53 %
SF	-41.87	5.13	-36.73	-5.08 %
Finland	-637.63	250.74	-386.90	-15.32 %
Ko	80.08	-257.86	-177.78	-76.65 %
Ka	-69.94	32.09	-37.85	-6.36 %
SP	-82.62	36.42	-46.20	-4.69 %
Russia	-72.47	-189.36	-261.83	-14.44 %
Es	9.88	-61.38	-51.50	-18.94 %
Total	-700.23	0	-700.23	-15.19 %

Table 3: Reduction cost figures without (column 1) and with transfers (column 3) (MFIM).

Finally, even if we could not formally prove that the transfers (3.12)-(3.19) induce coalitional rationality in the case where damage functions are nonlinear, we nevertheless verified this property numerically *for each period*: at each of the 28 periods we first determined the 121 possible coalitions of the type described in Section 3.1 and then recomputed the corresponding constrained equilibrium, starting from the situation obtained at that period. Thus *a total of 3388 constrained equilibrium calculations were performed to verify our hypothesis.*

4 Coalitional rationality when the initial situation is not a Nash equilibrium

4.1 The need for a new formula

The financial transfers and the related numerical experiments presented above are all based on the assumption that the initial state is a Nash equilibrium between the seven regions. This assumption may be unrealistic: for instance, it can be argued that the initial situation in our

[8] In this table (and in Tables 4 and 5 below), subscript 0 refers to the initial state and subscript * to the final state.

[9] This was already observed in Germain *et al.* (1996) when the $D_i(\cdot)$ are linear, but the relative decreases in cost are more important in the present context. As the absolute gains are smaller, this can only be explained by the fact that the initial costs are lower, due to the different parametrisation of the cost functions.

example (1987) is a Nash equilibrium between Finland (with its three regions merged) and the USSR (whose four regions are also merged), which is different from a Nash equilibrium between the seven regions. This alternative initial state was assumed by Kaitala *et al.* (1995) and Germain *et al.* (1996), for instance. It is thus of interest to examine whether the financial transfers scheme discussed above still ensures individual and coalitional rationality when the initial situation is given which is not the Nash equilibrium.

In order to investigate this question, we apply our stage-wise negotiation technique and compute the transfers (3.12)-(3.19) in our example, but considering now that the initial state is a Nash equilibrium between Finland and the USSR of 1987. This implies that the damage coefficients π_i have to be adapted to this new context, and we choose them as in Kaitala *et al.* (1995) and Germain *et al.* (1996)[10]. A first difficulty immediately appears in that the cost and damage values at the Nash equilibrium are not available for calculating the transfers, because the emissions E_i^N do not satisfy the constraints (2.6) for the first few negotiation stages. We thus have to modify the transfer formula to refer to a *information constrained Nash equilibrium*[11] instead of the pure Nash equilibrium, at least as long as this latter point lies outside the region where values of the costs and damages are known. The results of the resulting numerical simulation are summarized in Table 4.

One immediately sees in this table that only Northern Finland, Kola and Estonia gain from the international cooperation, while Central and Southern Finland, Karelia and St Petersburg loose, sometimes significantly. Thus, not even the individual rationality is preserved if we insist on applying the transfer scheme (3.12)-(3.19) with the alternative starting point[12]. The reason of this undesired effect seems to be that the transfer formula shares between the regions the benefits that would be obtained from moving from the Nash equilibrium to the Pareto optimum, but that this ecological surplus is in fact much higher than that effectively available. Indeed, the value of the total costs for each region at the Nash equilibrium significantly exceed those at the starting point, and a large part of the surplus has therefore already been "consumed" by the initial situation. In other words, $J(E^N) - J(E^*)$ is much higher than $J(E_0) - J(E^*)$. As a consequence, the inclusion of the larger surplus in the transfer formula induces a substantial inbalance of the regions's contributions, as may be seen by considering, for instance, the large contribution of St Petersburg to the transfers and the remarkable benefit resulting for Estonia.

[10] For completeness, they are also given in Table 8 of the Appendix. A Nash equilibrium between Finland and the ex-USSR supposes that each region takes into account the impact of its emissions on the other regions of the country. For Finland, this corresponds to the three conditions

$$C_i'(E_{i,0}) = \alpha_i + 2\beta_i(\bar{E}_i - E_{i,0}) = \sum_{j=1}^{3} \pi_j Q_{j,0} = \sum_{j=1}^{3} D_j'(Q_{j,0}), \quad (i = 1, 2, 3),$$

which allows the identification of the parameters π_1, π_2 and π_3. The situation is similar for the ex-USSR.

[11] This equilibrium is defined as the situation obtained if each region acts individually and rationaly, subject to the current limitations on information specified by the constraints (2.6).

[12] Note that a similar result is obtained for the one-shot game corresponding to the same initial situation without any information restriction. The difficulty is therefore not attributable to the dynamic nature of the game, which is itself caused by the progressive availability of information.

	$J_i(E^*) - J_i(E_0)$	$T_i(E^*)$	$J_i^T(E^*) - J_i(E_0)$	$\dfrac{J_i^T(E^*) - J_i(E_0)}{J_i(E_0)}$
NF	-530.34	244.13	-286.21	-24.57 %
CF	-40.07	67.41	27.33	5.89 %
SF	-32.78	95.07	62.29	10.79 %
Finland	-603.61	406.61	-196.58	-8.91 %
Ko	127.33	-310.13	-182.80	-106.39 %
Ka	-57.50	74.73	17.23	3.18 %
SP	-67.35	202.21	134.86	15.78 %
Russia	2.48	-33.19	-30.71	-1.96 %
Es	63.65	-373.42	-309.77	-344.11 %
Total	-537.06	0	-537.06	-13.90 %

Table 4: Reduction cost figures without (column 1) and with transfers (column 3) (MFIM), starting from the Nash equilibrium between countries (alternative starting point).

Not surprisingly, if we redefine, for a moment, the players to be Finland, Russia and Estonia, then the cooperation appears to be profitable to all countries, even if individual regions loose. This can be explained by the fact that the initial situation is then much closer to a Nash equilibrium between these three countries than to a Nash equilibrium between the seven regions.

Given the difficulties associated with the transfers specified by (3.12) and (3.19), we now introduce a new transfer formula, in which one avoids sharing the unavailable part $J(E^N) - J(E_0)$ of the ecological surplus $J(E^N) - J(E^*)$ between the players. More precisely, only the part of the surplus that is available is now shared, that is $J(E_0) - J(E^*)$. Given this important modification, we may then re-use the developments of Section 3 where we replace all difference formulae, which were defined with respect to the Nash equilibrium E^N (see the note following equation (3.2)), by differences *with respect to the initial state* E_0. One then has that

$$T_{i,t} = -\Delta C_{i,t} - \Delta D_{i,t} + \sum_{j=1}^{n} \delta_{ij,t} \Delta G_{j,t} + \mu_{i,t} \sum_{j=1}^{n} \Delta R_{j,t}, \tag{4.1}$$

as in (3.12), but where we have redefined

$$\begin{aligned}
\Delta E_{i,t} &= E_{i,t} - E_{i,0}, \\
\Delta C_{i,t} &= C_{i,t} - C_{i,0}, \\
\Delta D_{i,t} &= D_{i,t} - D_{i,0}, \\
\Delta G_{j,t} &= G_j(E_{j,t}) - G_j(E_{j,0}) \text{ and} \\
\Delta R_{j,t} &= \Delta D_{j,t} - D_j'(Q_{j,t}) \textstyle\sum_{i=1}^{n} a_{ji} \Delta E_{i,t},
\end{aligned} \tag{4.2}$$

and where the $\delta_{ij,t}$ and $\mu_{i,t}$ remain given by (3.19). Observe that, if one chooses $E_0 = E^N$, as in Section 3.2, then the new and the old transfer formulae coincide. Thus the results of Table 3 may be reinterpreted as produced by the new transfer formula.

4.2 Application to the example

As in Section 3 (equation (3.17)), we may deduce the individually rational property of our transfer scheme *with respect to the starting situation*, while (3.17) only ensured it with respect to the Nash equilibrium. This property guarantees that situations as that illustrated by Table 4 may no longer occur. In fact, applying the new transfer scheme on our example produces the results shown in Table 5. The corresponding emission and transfer levels are pictured in Figures 4 and 5, respectively.

	$J_i(E^*) - J_i(E_0)$	$T_i(E^*)$	$J_i^T(E^*) - J_i(E_0)$	$\frac{J_i^T(E^*) - J_i(E_0)}{J_i(E_0)}$
NF	-530.34	209.98	-320.35	-27.51 %
CF	-40.07	25.99	-14.08	-3.03 %
SF	-32.78	19.17	-13.60	-2.36 %
Finland	-603.61	255.15	-348.04	-15.77 %
Ko	127.33	-261.38	-134.05	-78.01 %
Ka	-57.50	31.23	-36.27	-4.85 %
SP	-67.35	47.88	-19.46	-2.28 %
Russia	2.48	-182.26	-179.78	-11.47 %
Es	63.65	-72.89	-9.24	-10.26 %
Total	-537.06	0	-537.06	-13.90 %

Table 5: Reduction cost figures without (column 1) and with transfers (column 3) (MFIM), starting from the Nash equilibrium between countries and using the new transfer formula (alternative starting point).

Several observations follow from these results. The first is that, as predicted by the theory, every region gains from international cooperation when the new transfers are used. The second is that the total volume of the transfers (in absolute) value is significantly smaller with the new transfers, reflecting the fact that they redistribute a smaller surplus. Finally, we note that the contributions of St Petersburg and Southern Finland have been significantly reduced, in parallel with the final gain of Estonia. In fact, the values of Table 5 are closer to those of Table 3 than to that of Table 4. This is reassuring as it indicates that the new transfer scheme is relatively stable with respect to variations of the assumptions on the initial situation, at variance with the formulation of Section 4, based on the proposal of Chander and Tulkens (1996).

Finally, we note that coalitional rationality of the new transfer scheme has not been theoretically clarified, as was already the case for the extension of the Chander-Tulkens transfers to nonlinear damage functions. However, we also tested this property numerically for our example, comparinging the potential benefits made by all possible coalitions of the type described in Section 3.1 to that obtained from full cooperation with the new transfers. This again required the numerical solution of 121 constrained equilibrium problems (one or each of the possible coalitions)

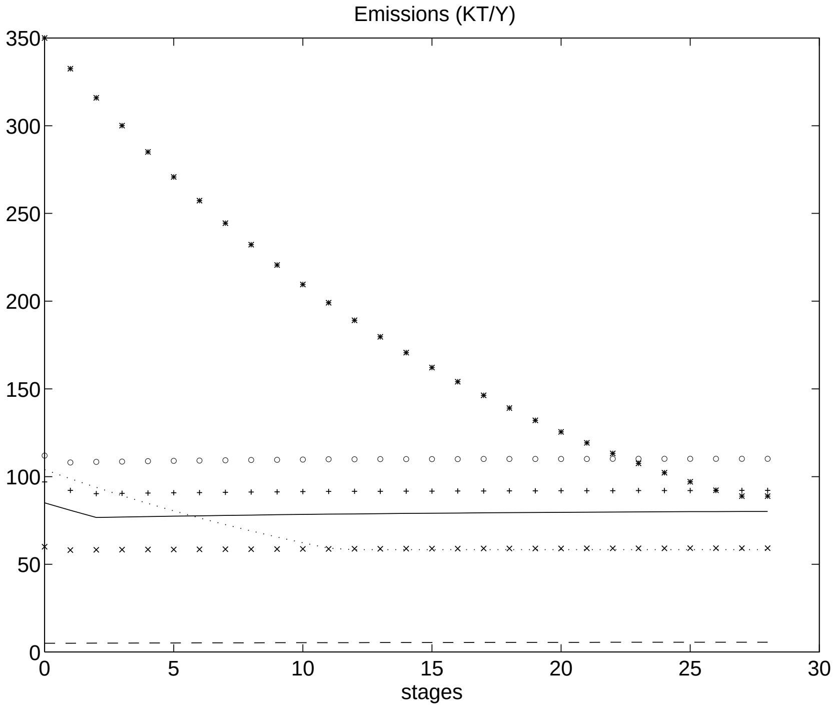

Figure 4: Evolution of the emissions per region (alternative starting point).

at each of the 28 periods, that is a total of 3388 such problems. In our example at least, none of the coalitions dominates the international optimum, a very encouraging result.

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6 Conclusion

Using a quadratic formulation of the cost functions, we have proposed and analyzed a dynamic cooperation process to reduce transboundary pollution between several countries or regions. This process ensures that the Pareto optimum is attained in a finite number of negotiation stages, each consisting of a period of negotiation on the emission reductions, based on limited information, followed by a period during which emissions are effectively reduced. Compared to Germain *et al.* (1996) where linear damage functions where considered in a similar context, we show that emission levels are no longer decreasing, even if their order of magnitude remains similar. We also investigated the calculation of financial transfers between the players, inspired by the proposal of Chander and Tulkens (1996), and indicated that these transfers make the procedure

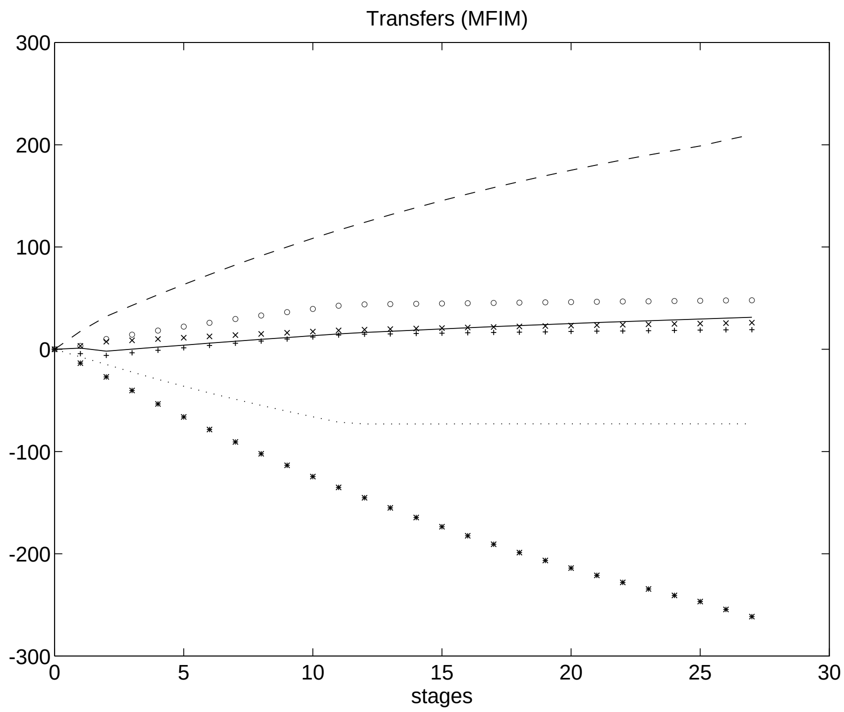

Figure 5: Evolution of the transfers per region (alternative starting point and new transfer scheme).

strategically stable if one starts from a Nash equilibrium. However, severe difficulties may occur with these transfers if the initial situation differs from a Nash equilibrium between the players. In an attempt to alleviate those difficulties, we introduced a new way to compute the financial transfers, whose structure no longer depend on the Nash equilibrium, but rather on the initial situation. We showed (in part only numerically) that these new transfers induce individual and coalitional rationality.

The authors are however well aware that the proposed results have their limits. An important one is that coalitional rationality has not been formally proved in general for any of the considered transfer schemes. It would be desirable to obtain this property for general strictly convex functions, or, at least, in a framework where emissions are no longer monotonically decreasing. However, the very definition of a new transfer formulation, which appears to be stable with respect to alternative initial situations, is encouraging, and opens interesting research perspectives.

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Appendix

i	Region	$E_{i,0}$	$Q_{i,0}$	B_i	α_i	β_i	γ_i	$\tilde{E}_i$	π_i
1	Northern Finland	5	46	26	10.0	2.093	5.9	5	1.09
2	Central Finland	60	98	59	3.8	0.172	33.0	60	0.09
3	Southern Finland	97	66	35	4.6	0.068	53.6	97	0.24
4	Kola:	350	131	27					0.02
	• $98 \leq E_4 \leq 350$				1.0	0.000	0.0	350	
	• $0 \leq E_4 \leq 98$				1.0	0.077	252.0	98	
5	Karelia	85	95	50	4.0	0.045	0.0	85	0.12
6	St Petersburg	112	88	46	6.0	0.051	0.0	112	0.22
7	Estonia:	104	60	32					0.05
	• $60 \leq E_7 \leq 104$				2.0	0.000	0.0	104	
	• $0 \leq E_7 \leq 60$				2.0	0.191	88.0	60	

Table 6: Parameters and initial levels

	Emitting region						
Receiving region	NF	CF	SF	Ko	Ka	SP	Es
Northern Finland (NF)	0.200	0.017	0.010	0.046	0.012	0.000	0.000
Central Finland (CF)	0.000	0.300	0.062	0.011	0.047	0.036	0.029
Southern Finland (SF)	0.000	0.017	0.227	0.003	0.000	0.027	0.038
Kola (Ko)	0.000	0.017	0.000	0.286	0.023	0.009	0.000
Karelia (Ka)	0.000	0.033	0.031	0.017	0.318	0.045	0.019
St Petersburg (SP)	0.000	0.017	0.031	0.003	0.012	0.268	0.058
Estonia (Es)	0.000	0.000	0.031	0.000	0.000	0.018	0.221

Table 7: Transport coefficients in 1987

NF	CF	SF	Ko	Ka	SP	Es
1.09	0.129	0.307	0.027	0.132	0.254	0.151

Table 8: Coefficients π_i for the case where the initial situation is a Nash equilibrium between countries (alternative starting point).