Transportation modelling methods and advanced transport telematics (ATT)

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Contents

1 Introduction: the context of traffic modelling

The traffic along your usual route on the motorway seems rather dense this morning, and you anticipate congestion when entering the city. What is happening today? The radio, chattering in the background, just informs you that a serious incident occurred about five miles further on your route and causes an important tailback. Taking this information into account, you immediately consider alternative routes from the next exit, that would (hopefully) avoid the congested area and bring you in time for your morning appointment...Your choice is rapidly made, and you soon recover your peace of mind, just marginally perturbed by the usual doubt on the validity of such decisions.

Who, amongst the crowd of daily commuters, hasn't experienced this scenario or one of its variants? Who did never think about the best route to work, to the kid's school or to the supermarket? More generally, who never wondered about the causes and behavioural rules that govern the flow of so many daily trips?

Although traffic modelling and analysis do correspond to this basic curiosity about the behaviour of fellow human beings, the true utility of these techniques really shows up when one realizes that they are absolutely necessary for defining transportation strategies at the level of society as a whole. And the necessity of such strategies can no longer be discussed today: to be convinced, just consider the staggering societal cost of the current transportation system.

- Around 55,000 people are killed every year on the roads on the countries of the European Communities only. A further 1.7 million are injured and 150,000 more are handicaped for life. The human and societal cost of these accidents just cannot be measured. Their purely financial cost exceeds 50 millions ECU per year.
- The financial cost of traffic itself is estimated to be around 500 billions ECU per year for the same countries, most of which is caused by congestion and inefficient route choice.
- Pollution caused by transportation (private vehicles, trains, busses, planes,...) is a serious concern. Its annual cost in Europe only is over 5 billions ECU.

These numbers, extracted from official EEC documents [1] reveal the urgent need for a better planning of transportation. This urgence is further reinforced by the commonly accepted forecast that transportation demand is likely to grow by about 3% yearly at least until 2000...

Understanding the transportation phenomenon in depth is therefore important. This understanding must ideally lead to tools (the transportation models) allowing traffic forecasts when the transportation network and/or demand and/or behaviour is modified. They first allow a better understanding of the process itself, but their principal use is for transportation *management*, both in its daily operation (traffic regulation), and for long term planning (strategic planning and investment policies). The many questions raised in these contexts are of course of extremely diverse nature. The group in which the author works has, for instance, studied the global impact of the displacement of a high school and an hospital on the traffic in a small city, the long term effects of ring roads, the efficiency of relief routes, the feasibility of automated urban transportation and the impact on congestion of variable message signs for incident warning, just to cite a few cases.

Quite naturally, transportation models are adapted to this diversity of application fields, but they all share a common purpose: that of making possible the establishment of strategies for improving comfort and safety of users.

The acutest transportation problems arise of course in big cities. Most of these indeed suffer from the negative effects of an extremely dense traffic: Paris, London, Athens, Rome, Barcelona, Brussels, ... are indeed faced (to a varying degree) with all the nuisances caused by daily commuting. The basic traffic modelling methodologies naturally appeared in this context, but are now the subject of a necessary re-evaluation. Indeed, the introduction of new information technologies in both urban and regional traffic has already modified the behaviour of many transportation network users in many cities. Frequent radio bulletins indicating congested areas and incidents are now part of the daily commuter routine, and have substantial impact on his/her transportation strategy. Further and more radical changes are afoot: route guidance techniques (autonomous or interactive), city map updating via radio beacons, variable message signs for traffic control and parking information, road pricing systems are but a few of the techniques studied and applied in the contemporary management of transportation networks. In Europe, international programs, as EUREKA PROMETHEUS and the EEC DRIVE effort, have been set up to analyze and optimize the use of these new information and control technologies (RTI[1] and ATT[2] in the DRIVE terminology) in a quickly evolving environment and society, but this line of research and development is also active in the United States and Japan, where similar programs are proceeding.

This evolution clearly questions the validity and relevance of classical modelling techniques. Indeed, the very purpose of these new techniques is to modify the road user behaviour in order to improve safety and relieve congestion problems in the network. It is therefore a crucial challenge that the traffic modelling techniques, now widely use for strategical transportation planning, can realistically predict their effect both in qualitative and quantitative terms. Another challenge for the traffic modeller is that the actual behavioural reaction of transportation users to the new information technologies is not correctly known, and will probably be the subject of continued research and refinement for the coming decenny.

We discuss in this paper the impact of those challenges on the classical traffic modelling methodologies described above. Some consequences of this discussion will be drawn, hopefully providing directions for further development.

The paper, which extends both [19] and [50], is organized as follows. In Section 2, we outline the basic concepts in transportation modelling. Section 3 is devoted to the exposition of the most important methods in this area, with some emphasis on the notion of equilibrium. We next examine in Section 4 some further limitations of transportation modele, before providing, in Section 5, a partial overview of some widely used packages. Section 6 is devoted to the new generation of behavioural models and Section 7 finally proposes some conclusions.

However, before we actually detail methodologies, their applications and limitations, it is important to discuss, albeit briefly, the context in which transportation modelling is actually used.

1.1 Political evaluation

The first major arena where transportation modelling is widely used is that of the political evaluation of transportation policies at city, regional, national or even European level.

As alluded to in the introduction, transportation problems arise in the everyday life of most citizens. Any policy with a strong impact on the daily "way of life" of so many voters clearly warrants, from the political point of view, a careful evaluation. Furthermore, as the decision issues become more and more urgent and complex, decision makers are less and less willing to take strong options on the basis of their understanding alone and rely on (hide behind?) expert

[1] Road Transport Informatics.

[2] Advanced Transport Telematics.

advice in a steadily larger degree.

We also stress here the intimate relationship between transportation planning and land-use regulations, another politically important subject. Most transportation experts are very well aware of this relationship, but it remains a difficult task to integrate both approaches in coherent, practical and politically viable policies.

1.2 Technological assessment

Transportation models are also used in technology assessment. Indeed, the technological development in transportation is often costly, which makes a sound economical evaluation of its effects and market crucial. The preoccupation is therefore different here from that in the previous paragraph: instead of considering the effects of transportation policies at a society-wide level, the impact of transportation related techniques is now considered from the point of view of specific actors (productors, implementation organizers) within the society.

The areas of transportation technique concerned by these considerations broadly fall in the following categories.

Infrastructural design. Probably the oldest field of application for transportation modelling is that of improvement and updating of transport infrastructures. Measuring the impact on traffic of network modifications, both in private and public transport, is indeed a very common application. The diversity of situations is nevertheless very large: motorways and urban roads, trains, busses, cars, cyclists, pedestrian routing, general infrastructure maintenance and all combinations thereof provide a wide domain of investigation.

Technological advances in vehicles. A number of applications have been made in relation with todays technological advances in vehicle technology. Started many years ago with the "cruising" features (mostly in american cars), the innovations giving the vehicle some intelligence and more acute perception of its environment have flourished. Gap detection and warning, automatic speed adjustment, synchronized driving all have their effect on the general traffic, not to mention the simple improvement in car performances.

Technological advances in communications. Possibly the greatest challenge in today's modelling arises from the advent of advanced information systems for transportation users. These developments are based on the premice that informing transportation users modifies their behaviour. The behavioural modification, at the user's level, is then reckoned to cause significant change in the global transportation scene at city-wide level.

Of course, the problem is then to infer these global changes from the sometimes rather detailed knowledge of the information distributed. Questions of interest are then related to the broadness of information distribution, its accuracy, reliability, timeliness and method of transmission. The attitude of users with respect to this information is also a much studied subject.

Regulations in the transport arena. Finally, the establishment of new regulations dealing with transportation also raises the question of their impact on traffic. One immediately thinks of parking policies, scheduling of goods delivery in city centers, road pricing or speed limits enforcement on motorways.

In many of these sectors, strong emphasis is put by the user on the reliability aspects of the involved models.

1.3 Environmental analysis

A last domain of application for transportation models is that of environmental analysis. The idea is that a number of nuisances generated by traffic are somehow proportional to traffic itself, and thus that traffic forecasts might help in predicting the evolution of these nuisances. Noise, gas emission and the related atmospheric modifications, vibrations and chemical effects of winter road salting are part of a set of preoccupations of growing importance in western societies.

This type of application is the subject of intensive study. It is complicated by the fact that the relation between pollution and traffic is often complex and not always fully understood: this leaves room for considerable debate and substantial methodological improvements.

2 The basics of transportation modelling

2.1 The spatial dimension

2.1.1 Zones and centroids

Like all models, transportation models considerably simplify reality. For instance, it is nearly always impossible to handle a level of detail where trips can be specified from building to building or from address to address in a city. It is thus customary to define *zones* that cover the considered area, the population and activity within these zones then being concentrated in a point *the zone centroid*. Trips are then considered from an *origin zone* to a *destination zone*.

It is important to note at this stage that the definition of these zones is intrinsically tied with the availability of data. Indeed, if one wishes to estimate the transportation demand from or to one of the zones, one must clearly know some of their characteristics: population, sociological structure, attractivity for various purposes and accessibility, for example. It is therefore frequent that the very definition of the zones explicitly takes the availability of these informations into account, or, at least, the cost and effort necessary for obtaining them.

Besides the problems linked to data collection, other factors also influence the definition of zones. The purpose of the study being made may also suggest some natural aggregation. For instance, if one considers the traffic perturbations caused by the opening of a new shopping mail, it is clear that the immediate surroundings of the mail should be modelled with a certain amount of detail, while a more and more aggregate description might be adequate when the distance to the mail increases. Depending on the application, a city as Brussels could count from twenty zones to more than a thousand...

2.1.2 The transportation network

Once the definition of zones established, it is then possible to specify the transportation network itself, as the set of links between these zones. There will generally be more than one subnetwork, each corresponding to a different *transportation mode*: roads for cars, bus lines, cycle lanes etc. Such a complex network is then called a *multimodal* transportation network.

As for the zone definition, different considerations influence the network description to be retained in the model and its level of detail. Availability of data about the network links, study purpose and integration of the network with the zone typically come into play. It is, in fact, rather fallacious to consider the definition of zones and the network specification as two successive and distinct phases: they are intimately linked to each other and should be integrated in a global and simultaneous approach.

The modelling of an urban area using these techniques is usually represented by a mathematical object called an *oriented graph*: *nodes* (points in the plane, for example) are linked together by "arrows" indicating the possible direction of travel (the *links*). Zones are then associated with nodes and network links to arcs.

2.1.3 Local models

The above paragraphs focussed on modelling transportation at the level of a city or a region. But this is not the only type of modelling and much more local models are also of importance. The typical situation is when one desires to analyze, forecast or control a limited set of connected intersections. In this case, the definition of the zone under study often has nearly immediate implications in the definition of centroids and links: links are given by the actual roads and possible turns within the zone, while centroids and nodes are associated to the entry/exit points of the zones and with the inner intersections.

2.1.4 ATT and spatial modelling

The impact of new advanced technologies in transportation has an important consequence on the spatial description of models. The main observation is that the object of study is no longer restricted to the movements of users in the transportation network, but also covers the exchange of information across its various parts: sophisticate route guidance or incident detection as well as relatively classical traffic light synchronization involve that exchange very clearly. Furthermore, the efficiency of advanced ATT is often said to be dependent on the quality, reliability, accuracy of the exchanged information, and on its distribution method.

Typical examples of relevant factors are

- the location of traffic monitoring equipment (the actual locations of detection and measurement equipment on the network of course conditions the final information in that it specifies the parts of the network about which information could be available),
- the location of information distribution mechanisms (conditioning in turn the locations at which re-routing can occur),

- the accessibility of information (defining the classes of network users accessing possibly different types of information),
- the strategies used to process information from the network before subsequent redistribution to users,
- the actual interconnection pattern (topology) of monitoring devices, information processing centres and distribution mechanisms.

Besides the classical transportation network, we therefore discover another network, which describes the information flow (the feedback loop) between the transportation network and its users. We call this network the *information network*. It is the author's view that explicit modelling of this information network is an important part of the present research challenges in this area.

Of course, these information network models should be flexible, in order to adapt to the many different cases of potential interest. In particular, they should include the relevant factors mentioned above, but also parameters that influence their efficiency (communication links' or detectors' reliabilities, for instance) and thus also permit the modelling of information system breakdowns. Information network models will be further discussed below, but their very appearance on the spatial modelling scene is crucial to note at this point.

2.2 The time dimension

2.2.1 Static and dynamic transportation models

A fundamental distinction appears at this stage between two distinct model classes: *static* and *dynamic* models. As could be inferred from these terms, both approaches differ by the way in which they consider the traffic evolution in time.

Historically the first, *static models* describe traffic on a time period that is long enough to ensure that all travellers starting a trip in this period also reach their destination within the same period.

One therefore obtains a considerable conceptual simplification, because it is no longer necessary to analyze traffic on the network links at successive moments. It is indeed sufficient to consider the cumulative value on the considered time period. This simplification is of course made at the expense of some realism, but the results obtained with this approach can however be adequate for some kinds of studies. (As the wear of road surface is roughly proportional to the total volume of traffic on the road, but is independent of its repartition in time, it is thus enough to know an approximation to this total volume for establishing a maintenance program for the city roads...).

Dynamic models explicitly introduce the passing of time within their mechanism. These models originate from the deficiency of static models in correctly describing route choice in peak period, where the traffic on the network links can significantly differ from its average value on the day, say. Moreover, not only the routes but also transportation modes and departure times can vary: dynamic models are thus *necessary* to describe transient traffic phenomena, as congestion experienced by commuters on the big cities' approaches.

Practically, these models are much harder to build because the quantity of needed parameters and results is usually much larger than for static ones. It is indeed necessary to make every quantity fully time dependent. The requested data collection as well as the a posteriori validation are therefore much more complex.

As most models for time dependent phenomena, dynamic traffic models can be classified into three broad categories, depending on the method they use for representing the passing of time.

- Continuous models, first, consider traffic parameters as continuous functions of time, and they aim at determining some of these functions (as traffic density, for instance) as a consequence of other such functions. It is in this context that Lighthill and Whitnam developed in 1955 [37] their famous model for a phenomenon well known by many road users: the "moving traffic shock waves" delimited by zones of slowing and zones of accelerating traffic.

- The discretized models, on the other hand, describe the time dependence of traffic parameters by representing the time in successive "slices" during which the transportation network has a constant behaviour. The studied effects (as congestion) therefore vary from one timeslice to the next. They are often calculated from laws that link their value at a given timeslice with other values from preceding timeslices.

  This technique has the further advantage to allow a progressive transition between static and dynamic models. A static model can indeed be considered as a dynamic model with only one (long) timeslice. The model is then made "dynamic" by increasing the number of timeslices and reducing their length. One then obtains mixed models whose behaviour is similar to that of static ones for long timeslices (from 0 to 6am, for instance) but still allow for a finer time analysis at peak periods, when timeslices are shorter.

  Discretized dynamic models are also closest to *dynamical systems* in the sense of the mathematical theory of chaotic movements: the way the model behaves at a given step only depends on the few previous steps of its history, and its evolution continues from stage to stage without any reference to a steady-state.

- A third way to handle the passing of time in models is the so-called "event driven" technique. Time is considered as a sequence of events, each of them implying some modification in the state of the model, with the possibility for an event to define one or more other events in its future. Such events might be traffic lights changes, arrival of vehicles at intersections or bus-stops, a traveler's departure or arrival, network incidents or information transmission. This technique is thus especially adapted in the treatment of transient phenomena, since these are mostly caused by such "events" in the network, and to models involving some type of incident detection and warning.

  Another advantage of this methodology is that it is most suited to exploiting parallel computer architecture, whenever available. Different events in different geographical areas can indeed be handled in a completely asynchronous fashion by different processors. This feature is far from being anecdotic, because the computational requirements of dynamic models is in general substantial, and techniques to speed processing up are very worthwhile in the framework of real time applications.

The distinction between continuous and discretized models is further blurred in practice because continuous models often use numerical techniques that rely on an internal time discretization: timeslicing is thus implicitly present even in purely continuous approaches. Similarly, dynamic models with time-slices may also be considered as "event-driven", where the only events are associated with the end of each time-slice.

2.2.2 ATT and the passing of time

Because much of ATT is geared at the dynamic behaviour of transportation users and at the control or analysis of transient phenomenons, dynamic transportation models seem unavoidable within this context. Network users indeed have clearly different strategies in peak hour as opposed to off-peak, in week-ends as opposed to week days, in day as opposed to night, and the list of examples can easily be made longer. Furthermore, techniques like intelligent incident detection and interactive or automated route guidance are only meaningful in a dynamic context.

It is remarkable that, although appreciated by many experts, this conclusion has not always been applied in its full requirements. Because of the difficulty in building intrinsically dynamic models, static models whose parameters were made dependent on time have often been used. Whether this methodology is in fact coherent is clearly questionable, and heavily depends on the other factors in the chosen methodology. The crucial question is in the manner in which information (traffic or other) is propagated along the time dimension and the assumptions made on this propagation. Parts of this important subject will be discussed below in relation with equilibrium models. But modellers and model users interested in ATT should bear in mind that it isn't usually enough for a model to depend on time to become a truly dynamic model.

2.3 Scenarii and the reference case

Once a transportation model is built, it is then calibrated to represent the true nature of the situation under study. This calibration is usually carried out by comparing the model's output to otherwise known measures of the real situation. It is therefore important for a modeller not to use all the information he has at his disposal to build the model, but to keep some of it for this calibration phase.

Once calibrated on the real situation (often called the *reference case* or the *do-nothing case*), the model can then be used to analyze diverse alternative scenarii, differing from the reference case in well defined manner. The confidence one has in the obtained results for these alternative scenarii is clearly conditioned by the quality of the model calibration on the reference case.

Playing with different scenarii and using the model in this context then helps building an understanding of the situation and an appraisal of the scenarii themselves. It is the author's belief that this *cognitive use* of transportation models, with the main emphasis on understanding the qualitative effects of the scenarii, is most appropriate in many circumstances, as opposed to a simple *predictive* use in which the scenarii are quantitatively simulated but without a posteriori analysis and synthesis of their inner mechanisms.

Of course, the definition of the reference case and scenarii is crucial in the proper use of models. This is especially true in the presence of ATT, where more subtle questions must be settled. For instance, an important and not so trivial question is to define the reference method for information distribution and propagation across the information network and the transportation network itself.

2.4 A paradigm for city-wide modelling: the four stages model

2.4.1 The classical method

Once the decisions about the territory and time representation taken, the model is already well shaped: it contains zones and a network for travelling from one zone to another. We should then use this structure for describing the traffic itself. We now focus our attention to a very classical methodology for city- or region-wide modelling originally proposed by Wilson in the beginning of the 70ies [55]. This method proceeds in four successive stages:

Traffic generation. In the first stage, each centroid of the model is considered and the number of trips leaving the associated zone, a well as the number of trips ending in the zone are determined. This is the *generation stage*.

This stage classically uses statistical techniques for predicting the "emissivity" of a zone, that is the number of trips that depart from that zone. These techniques are based on the repartition of inhabitants (households) in coherent socio-economical groups, whose displacement behaviour can be analyzed. Similarly, the "attractivity" of a zone, i.e. the number of trips ending in the zone, can be establishes for various causes of displacement (the *functions*: work, shopping, school, etc.). Data is typically obtained from special surveys.

Traffic distribution. The second stage then consists in calculating the repartition of these trips in (origin, destination) pairs. One speaks of the *distribution stage*, whose result can be placed in a big table: each entry of the table contains a number of trips leaving the zone corresponding to the row of the table and ending in the zone corresponding to its column. This is the well-known *origin/destination matrix*, or O/D matrix.

Modal choice. The third stage models the competition between the various transportation modes (private cars, train, bus, metro, cycles,...) and establishes, for each (origin, destination) pair, the proportion of the trips using the different modes.

Traffic assignment. Finally, the fourth stage, called the *assignment stage*, determines the actual routes in the network that are being used for each of the trips and each of the modes during the considered modelling period.

This four stage methodology has often be criticized because of its sequential character. For example, the choice between car and bus for a trip may depend on the congestion level on the roads or in the public transportation system, level which is not known until after the fourth stage. Congestion can also encourage drivers to change their destination, or even, in extreme cases, to forego their trip altogether. More generally, the four *successive* stages are often integrated and interdependent in practice.

We note that we haven't specified typical methodologies for the last three of the four stages. A survey of these methodologies is the object of the next section.

We finally note the the four stage methodology can also be applied, even partly, in more local studies, like the analysis of a set of intersections. In such a case, one would typically estimate the O/D matrix from measurements, say, which correspond to Stage 2, and then assign the corresponding demand on the considered network, which correspond to Stage 4.

2.4.2 ATT and the four stage model

Unfortunately, using the four stage modelling approach in an ATT framework is clearly a questionable methodology. The inherent sequentiality of the technique is here completely unrealistic, in the measure where information flow from one of the four stage to previous ones is very common in the presence of ATT. For example, home-based information systems on expected travel time using private or public transportation are designed to modify the modal choice (Stage 3) as a function of network performance (Stage 4). Congestion dependent road pricing schemes have a similar effect, but are also anticipated to alter the very transportation demand (Stages 1 and 2).

Another important aspect of this global approach in the context of ATT is the necessary dynamic character stressed above. Although a number of dynamic models have been proposed for traffic assignment (Stage 4), very much less is known about dynamic city-wide demand models, and even less on the necessary integration of both. This is probably the area where transportation modelling in the ATT framework needs improvement most. The situation is less critical for more local studies, as there exist methodologies for estimating a dynamic demand in this case (see below).

3 Methods in transportation modelling

We now examine some classical methods and concepts whose application in transportation modelling is very widespread. We also propose, case by case, a short criticism of their relevance and adaptability in the ATT framework. Of course, this discussion does not cover all ideas that have been suggested in the sector. Some further ideas (in Section 6.2) will also be considered as potential answers to the criticism on these classics.

3.1 User's rationality and utility functions

One of the most important question in modelling human behaviour is to decide on what is considered to be the "motor" or driving force behind the choice of a particular action. This very central question will probably always be open for debate. However, practical models have to make some kind of choice at this level. The main dimension along which existing methodologies can be ranked is the assumption that transportation users make, at least to some degree, *rational choices*. It is commonly assumed that using a transportation network always implies a certain costs for the user (economists talk about "disutility"): not only travelling takes time, but it could also cost in several other ways. The ticket price for the bus or the metro, the petrol consumed by our vehicle, the number of stops along the route, the accident risks, the inconvenience and stress caused by congestion...can also add to the mere time spent to our perception of transportation costs. Society as a whole also supports other costs associated with transport, as pollution, security, insurances, medical care, noise and other nuisances caused by traffic.

3.1.1 Cost functions

In transportation models, this notion of global cost associated with displacement is typically taken into account by the choice of a *cost function* or *performance function* that determines, for each link of the network, a "cost" as a function of the traffic on that link. Units for this cost vary from model to model, and are chosen in relation with the final purpose of the modelling. For example, the quantity of exhaust gas of a certain type can be used if one wishes to measure the pollution impacts, while transportation time can be used if one is interested at asserting the level of service of public transport...There is however a remarkable consistency between all these choices, irrespective of the units chosen: the cost function is typically increasing as traffic along the link increases, slowly first but faster and faster for higher traffic levels. For example, the travel time along a link is relatively independent of traffic for low traffic levels, but can increase substantially for more congested levels, up to the point where not a single vehicle can move because everybody is stuck in a traffic jam!

It should be noted that the choice of a single performance function associated with the parts of the transportation network implies a complete and uniform rationality of the users in the measure where each users minimizes his or her disutility. This assumption has often been criticized as irrealistic. One way to overcome the difficulty is to introduce some degree of stochasticity in the users' choice criteria. We will briefly discuss these aspects below.

Finally, optimizing a single "utility function" (even containing some stochastic component) often raises the question of comparing various goods or services of qualitatively different nature, a very difficult task. For instance, it is often necessary to estimate the monetary value of time (VOT) for users or users' classes. Some concepts in multi-objective optimization could present an interesting alternative at this level, but this approach is not widely used (see [45] for an example).

3.1.2 ATT and utility functions

The use of utility function within the ATT framework has its supporters and detractors. The main arguments in favor of this approach are

- the possibly high disaggregation level allowed by the methodology,
- the easier calibration of the resulting models (at least for moderate disaggregation levels) compared to other alternatives,

- the existence of good related mathematical and software tools.

Arguments that have been used against the technique are as follows.

- The behaviour of humans cannot be safely represented by the optimization of a utility function, mostly because many users do not know their utility function and also because they are, in complex cases, mostly unable to perform a proper optimization, especially when the decision process is quick as it is in traffic.
- The variation of perception among users' classes is badly represented by assuming random variations.

These considerations have led to modelling systems where behaviour is not represented anymore by the optimization of utility function, but by application of adapted *behavioural rules*. These rules are not assumed to reflect the (undetermined) rationality of the user, but are just considered as *descriptive* of behaviour, as opposed to resulting from deeper processing. This kind of approach is clearly very inspired by artificial intelligence techniques, where the inadequacy of utility functions has often been pointed out. This rule-based paradigm is very attractive in many ways (at least to the author) but present the difficulty of the necessary statement sufficient and meaningful rules, a very difficult task.

Of course, all these arguments have their weight, and it is indeed very difficult to judge in abstracto. A lot depend on the necessary level of disaggregation of the problem under study, and the modellers have to exercize their best talents to measure the impact of their methodological choice on the realism of their conclusions. But the issue is sometimes crucial and cannot be ignored.

3.2 Aggregation level

3.2.1 Micro-, meso and macro models

There are of course various ways to establish a cost function of the type that we discussed above. One of the first was proposed in 1934 by Greenshields [26], and measures the travel time along a link. It is based on the decreasing linear relationship between traffic speed and traffic concentration (the speed varying from that of a single vehicle on the link to zero for a concentration corresponding to a traffic jam), The travel time can then be obtained from the reciprocal of the speed. Techniques based on this cost function are still used today [36]: they are often called *macroscopic* because they consider traffic globally as a flow, without differenciating between its components (vehicles, travellers). The *microscopic* approaches, on the other hand, go into much more detail and analyze traffic on a link by simulating the behaviour of individual vehicles and their interactions: the speed of such a vehicle is assumed to depend on the speed of the preceding vehicle and on the gap between them. Some approaches, as that of the Karlsruhe school [54], even consider the psychological thresholds in the user's perception of these parameters. Microscopic methods usually allow a finer analysis of transportation costs, but at the expense of a much larger computing effort. The whole range between very aggregate and very disaggregate models has been explored, for instance by grouping users or vehicles in *packets* or *platoons*. Models using such intermediate aggregation level are often called *mesoscopic*. Today's trend is towards rather disaggregate models (see [8] for a discussion of this trend in various parts of the world).

At this point, it might be worthwhile to pinpoint a very common myth among transportation model users, namely that a greater level of detail (especially in micro-models) is always preferable from the point of view of accuracy and realism (see [40]). The truth is that several models use a number of details, in the road infrastructure for instance, mostly to reassure their users about the fact that "nothing has been overlooked". But the accuracy of the methodological assumptions on the other parts of the model (in areas like users' behaviour or demand analysis, just to cite two) very often makes the modifications resulting from the explicit consideration of these details totally marginal. As a conclusion, disaggregation is not an improvement in itself, but only in the measure where it is coherent with *all* modelling assumptions.

3.2.2 ATT and the aggregation level

The specific constraint on aggregation level appearing in the ATT context is that of behavioural coherency. As is clear from above discussions, the modelling of the transportation system in this context imposes that special care is given to user's behaviours, both in the short term (in immediate decision making like in route choice) but also in the longer term (attitudes). It is therefore important to base the models on a metaphor that provides this behavioural coherency.

It is quite difficult to think of a behaviourally coherent transportation flow, for instance, because the idea of flow implies some kind of uniformity in the pattern, leaving no room for differences within the flow. Although one could think of models describing the flow of several fluids together, this remains, to the author's knowledge, rather speculative. One is therefore naturally led to more disaggregate metaphors, involving groups of users with similar behaviour. Many such metaphors have been proposed at a meso level, as *platoons* or *packets*. They are adequate for ATT sensitive models in direct measure of their behavioural coherency. The situation is finally most complicated when considering ATT sensitive public transportation modelling, since, in this context, the only reasonable "unit" seems to be the individual user, which directly leads to micro-models.

As a consequence of this discussion, it seems that, in first approach, micro-models are more adapted to behaviourally coherent simulation. Unfortunately, micro-models tend to be more limited in spatial coverage, because of their intrinsic higher complexity. There is therefore a need to develop meso-models whose spatial coverage can be much wider but whose behavioural coherency has to be specially emphasized. One such modelling tentative is outline in Section 6.2.

3.3 Physical analogies

The first generation of concepts in transportation modelling is widely based on analogies with the physical world. Traffic is considered as a flow (see above) or an electical current. Various methods have been proposed to exploit these analogies. We will briefly describe here two "physical" concepts that are still of major importance in methods for traffic distribution, the second stage of the classical four stage model.

The first idea is that transportation demand between two centroids might be modelled using the law of *gravitation*, in which the intensity of the attraction force between two bodies is proportional to the masses of these bodies and inversely proportional to the square of the distance separating them. Transposed in transportation demand models, the trip demand from a given centroid i to another j is then proportional to e_i, the "emissivity" of i, and to a_j, the "attractivity" of j, and also proportional to some decreasing "deterrence" function f of their (shortest) distance d_{ij}. More formally, the numbers of trips q_{ij} between centroids i and j is then approximated by

$$t_{ij} = k\ e_i a_j f(d_{ij}). \tag{1}$$

In the strict gravitational framework, one chooses $f(d_{ij}) = 1/d_{ij}^2$.

Another somewhat similar idea is to use the concept of *entropy* in a physical system. In this approach, the distribution model attempts to recover a particular trip distribution such that its likelihood is maximized amongst all equiprobable demand configurations. An easy mathematical development then shows that

$$t_{ij} = k\ e_i a_j e^{-\sigma d_{ij}}, \tag{2}$$

where σ is a free model parameter. We immediately note the strong link between the entropy and the gravitational approach (see [10], for instance, for a discussion of this relationship).

One then typically distinguishes (see [49]) between "singly constrained" distribution models, where the emissivity of all centroids is assumed to be known, and "doubly constrained" models, where constraints are put on both emissivities and attractivities. The choice of a suitable model for traffic distribution might be a delicate question (see [49]). For instance, it often reckoned that singly constrained model are more appropriate for short-term studies or for studies involving non-work functions.

It is not clear today how to extend these methodologies in a dynamic behaviourally coherent direction. This is, in the author's opinion, one of the major difficulties in large scale ATT sensitive demand modelling.

3.4 Activity chains

There is however some hope that another methodology, *activity chain* modelling, will help in the development of inherently dynamic and behaviourally sensitive demand analysis tools.

The notion of an activity chain is nearly self explaining: this is the sequence of activities planned for one or more individuals of a household at a given time in the day and week. Given such chains (the more one knows the better), it is then possible to deduce the transportation demand by considering which successive activities do not take place at the same localisation. The examination of many such activity chains then leads to an intrinsically dynamic and behaviour orientated demand modelling, resulting in more classical dynamic O/D matrices. The advantages of this approach are mainly

- its intrinsic dynamic nature,

- its very adaptable framework (it covers, for instance, multipurpose trips, a common reality but gaping hole in many other modelling approaches),
- its sensitivity to behavioural variations across households, because different schedulings of the same activities are possible.

Of course, this technique is not without its drawbacks.

- It requires a large amount of data (travel diaries, mostly) which is often difficult to obtain.
- The structure of the activity chains is not always known a priori, which introduces yet another level of modelling, that of activity scheduling.
- Its practical treatment by reliable software tools is still in infancy.

Despite these serious inconvenients, activity chain modelling remains one of the most promising technique in the framework of ATT. The proceedings volume [31] presents an interesting survey of this research area.

3.5 The assignment of traffic

We now turn our attention to some more mathematical techniques that are mostly used within the fourth stage of the general model described above, that is the assignement of traffic within each modal network, given the demand.

3.5.1 Shortest paths

The first popular idea is that network users choose the route that is *shortest* (in time or in any more general cost measure), while assuming that the cost associated with each link is constant, i.e. independent of the traffic on that link. This technique has the advantage of being readily interpretable and also very efficient from the computational point of view. One indeed knows very efficient algorithms for computing shortest paths in a network, even when the number of nodes and links is very large (over 100000 nodes is quite feasible). Traffic assignment based on this idea is known as the *all-or-nothing* assignment, because all trips between a given (origin, destination) pair are assigned to the same single route, a rather crude approximation.

3.5.2 Wardropian traffic equilibrium

Probably the most famous and most commonly used assignment method is aimed at improving this approximation by assigning trips to more than one route between a given (origin, destination) pair. It is based on the notion of *traffic equilibrium*. Following the famous statement made by Wardrop in 1952 [52], "travel time on all used routes is smaller or equal to that that would be experienced by a single vehicle on any unused route". In other words, no network user can, in an equilibrium situation, improve its trip cost by a unilateral decision. One easily sees that a consequence of this assumption is that all used routes between an origin and a destination have the same (minimum) cost and that all routes of higher costs are unused. This concept is

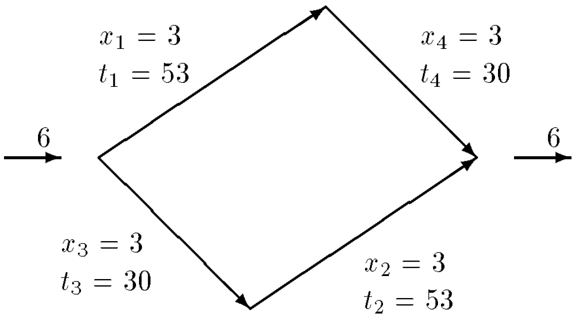

Figure 1: The initial equilibrium situation

obviously substantially more complex because it implies that the choice of a route between two zones depends not only on the possible routes between these two zones, but also on the load of the network, resulting from the traffic between all other O/D pairs! This concept is called "User Equilibrium" because the criterion used is the travel cost for the network user. The work of Beckman, McGuire and Winston [6] was amongst the first to use this principle in a very long line of research (see [15], [22], [23], [25], [48], or [49], for further discussion and references). One can also consider the "System Equilibrium" which would occur if one chooses all routes to minimizes the total transportation cost for all users (or for society). These two equilibrium concepts are unfortunately different, because the advantage of all may require that some users deliberately choose a route that is less favourable for them...The (alas) unrealistic nature of this assumption implies that User Equilibrium is most often used for describing the behaviour of users in the network, while the System Equilibrium is sometimes computed to obtain an "ideal" traffic situation.

3.5.3 The Braess paradox

The difference between User and System Equilibrium is best illustrated by the famous Braess paradox (see [12], [24], [16], [27], ...). Consider the simple network shown in Figure 1, with 4 links numbered from 1 to 4. Assume furthermore that the cost functions associated with these links are given by

$$\begin{aligned} t_1(x_1) &= 50 + x_1, \\ t_2(x_2) &= 50 + x_2, \\ t_3(x_3) &= 10x_3, \\ t_4(x_4) &= 10x_4, \end{aligned}$$

where the symbols $x_1, \ldots, x_4$ respectively denote the traffic flows on links 1 to 4. Assume finally that 6 users wish to go from the origin to the destination. This goal is achieved if 3 users choose the route (1,4) and the other three the route (3,2). It is then easy to verify that the cost associated with both these routes is of 83 units. Since there is no unused route and since all used routes are the same cost, this is a traffic equilibrium situation, whose total user cost is thus 498 units.

Suppose now that the city administration decides to improve the transportation network by

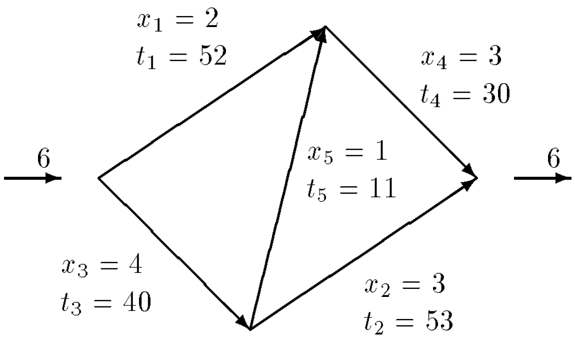

Figure 2: The intermediate situation

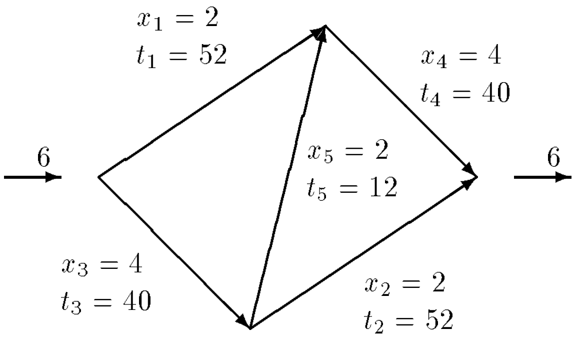

Figure 3: The new equilibrium

building an additional link (link 5) as shown in Figure 2, whose associated cost function is given by

$$t_5(x_5) = 10 + x_5,$$

therefore creating a new possible route (3,5,4). As soon as it is open, the users realize that the equilibrium is broken because this new unused route has a cost of 70 units, which is lower than that of the used routes. One user previously using the route (1,4) therefore decides to use (3,5,4) instead. We then obtain the situation pictured in Figure 2, where the cost associated with link 5 has now increased from 10 to 11.

But the route (3,5,4) is still less costly than (3,2), and one of the user of this route decides to switch to the new route, yielding the situation presented in Figure 3. It is easy to check that it is again an equilibrium situation.

The interesting point is that, despite the increased offer (the new link), the cost supported by each user has gone from 83 units to 92, and the total users cost from 498 units to 552! The network extension therefore has a negative impact, which can, at first sight, be considered as a paradox. But the "paradox" is of course only caused by the confusion between system optimality (which the city administration has tried to improve) and user equilibrium principle applied by the users.

Generally speaking, the Braess paradox indicate the importance of an in depth analysis of planned changes in the transportation network. It also shows, a contrario, that traffic restrictions (like pedestrian zones, one-way streets or road pricing) can have a beneficial impact on the global situation.

3.5.4 Equilibrium extensions and simultaneous models

It is important to note that the equilibrium method has been extended to handle situations more general than pure traffic assignment: one can talk of "equilibrium" between various transportation modes or of "equilibrium" between transportation demand and transportation supply (as specified by the network). The first of these extension typically requires a modification of the equilibrium concept, because the cost of public transportation cannot in general be compared to that of using a private car. Similarly, entropy ideas are usually introduced in the second extension for improved realism. We observe that both these approaches answer, albeit in part, to the criticism on the excessive sequentiality of the for stage model, because two or more phases are now interdependent and solved for simultaneously.

The reader is referred to [49], for instance, for a further discussion of the many extensions of the equilibrium methodology, including that of stochastic equilibrium that we introduce below.

3.6 Discrete choice techniques

Another powerful tool for transportation modellers is the theory of *discrete choices*. The main idea here is to assume that transportation users, when faced with a decision, can only choose between a limited set of possibilities, hence the name of discrete choice. With each of these possible choice is associated a utility function, and it is assumed that the user maximizes his utility.

The difficulty arises from the fact that these utility functions are usually not accurately known and cannot be measured easily. Moreover, the factors (attributes) that influence these utility functions (such as individual variability in sociological status, history, etc) are also beyond a complete analysis. To circumvent this problem, the utility functions are then constructed such that they contain a random part, which is supposed to represent the contribution of all these unknown factors. One then talks about *random utilities*. A random utility then merely gives the probability that a given choice will be made by the considered user.

The first major discrete choice model is the *logit model*, in which the probability of chosing a given alternative k among K possibilities is given by

$$p_k = \frac{e^{d(k)}}{\sum_{i=1}^{K} e^{d(i)}}, \tag{3}$$

where $d(i)$ is the deterministic part of the utility associated with the i-th choice[3]. As can be seen from this formula, this (sometimes irrealistically) assumes independence of the possible choices.

[3]This formula is based on the assumption that the random terms in each utility function are independently and identically distributed Gumbel, variables. The Gumbel distribution is an analytic approximation of the normal distribution.

The logit model is very widely used in all stages of transportation models: the modal choice is often represented by a logit model and the choice of reasonable routes (see below) can be made using this approach as well. The popularity of the logit model is strengthened by its ease of use in multinomial cases, that is when the number of possible choices for the user exceeds two. In this case, one often "cascades" the choices as a sequence of successive decisions, each of which being modelled by (3). One then talks about *nested logit* modelling.

An alternative model, the *probit* model, is based on the more general assumption that the utility functions are normally distributed but not necessarily independent. This model is however more difficult to implement because it leads to difficult numerical questions and also requires additional information on the choices (the covariance matrix), compared to the logit approach.

The reader is referred to [9] for a comprehensive coverage of discrete choice theory in the framework of transportation modelling.

3.7 Least squares fitting

We close this overview of the methodologies used in transportation research by mentioning that, as is the case in many scientific applications, one often tries to fit a given model to a set of observations. For instance, one might try to fit a OD-matrix describing the transportation demand to observations of traffic flows or centroids' emissivities. Among the many methods for observation fitting, the least squares approach is often preferred (see [13] for an example of application to the distribution stage).

One should also mention here that some inherently dynamic methods for local demand analysis can also be viewed in the framework of the least squares approach, in that they try to exploit correlations between flows penetrating into and leaving the study zone (see [14], [33] for instance).

3.8 ATT and traffic assignment

3.8.1 The equilibrium concept within the ATT framework

Many classical equilibrium models assume a fixed "design period" in which all trips are supposed to happen and during which congestion in the network is stable. This assumption simplifies the route planning decision as only one state of the network needs consideration, which has obvious computational advantages. However, this assumption has often been criticized (see [29], for instance) on the basis that trip timing may itself be dependent on the network congestion. Without diminishing the importance of this remark, we also note that a further difficulty is inherent to this type of equilibrium models for a situation where real time driver information systems are used: because they compress the whole route decision and trip making in a single moment, no information can reach the driver *during the trip* and no subsequent re-routing is possible[4]. This remark is also valid for "pseudo-dynamic" models that proceed by assuming

[4]It is fascinating to note that some computational methods to compute static equilibrium (see [49] or MICROTRIPS, for instance) proceed incrementally by successively reassigning flows using congested arterials to less congested ones: the iteration procedure in effect simulates the passing of time until convergence occurs to a time independent steady-state result.

equilibrium in each one of a succession of time slices (as the recent "dynamic" extension of TRIPS). Hence, the only information systems that can be modelled using these techniques are systems that influence driver behaviour (route and mode choice) before departure. This limitation is extremely serious, as it rules out a number of the techniques that are currently practically envisaged, including interactive and autonomous route guidance, advanced incident detection and many applications of variable message signs. We therefore conclude that such models are not well suited to our purpose.

It is interesting to note that we had to dismiss static equilibrium models mostly because of their static nature. One could therefore hope that *dynamic* equilibrium models would be more appropriate. This class of models evolved from the criticism mentioned above concerning the static models. Developments in this area are presented or reviewed in [7], [29], [39], [42] [53] [56], and [57], to cite only a few. They introduce the time dimension and are based on generalizing the notion of Wardropian equilibrium in time: the network must be "in equilibrium" at all moments during the design period (see [53] for a detailed formulation and discussion of this extension). Despite their increased computational cost, these models are conceptually attractive, because they provide the explicit time reference that is missing from the purely static approach. We therefore examine their underlying assumption (equilibrium itself, as opposed to mere static equilibrium) for consistency with the aim of modelling advanced driver information systems in the ATT context.

If we again analyze our statement of equilibrium, we note that a used path has its journey time compared to that "which would be *experienced* by a single vehicle on *any* unused route", where we have highlighted two very important words. The fact that journey time along unused paths must be "experienced" implies that the network users have to acquire that experience and knowledge. This can be achieved in two ways:

1. They acquire it by actually trying these "unused" non-optimal paths.

   This has the unfortunate consequence that the equilibrium principle becomes ipso facto invalid. However, the amount by which it must be violated might be quite small: most drivers can acquire their knowledge of the network over long periods and only use optimal routes in the "steady-state" of the system. The fraction of "exploring" drivers may then be neglected. This however implies that the experience accumulated over time must not be too quickly outdated. This situation can therefore only occur in transportation networks whose load is very *stable* in time.

2. They acquire it from some information system(s).

   This second branch of the alternative does not require stability of the transportation network characteristics, but then implies that the equilibrium state intrinsically depends on the quality of the information system(s) considered. This remark is strengthened by the observation that the equilibrium principle also assumes that the journey time of *all* unused paths is known. Equilibrium in the classical sense therefore requires *complete* and *accurate* information to be available for all network users at *any moment* during the design period

Furthermore it also assumes that these users have the capacity to correctly process this information in real time.

Thus we see that the very notion of traffic equilibrium can only make sense if either the traffic situation is not varying much over long periods or if some network-wide oracle provides every user with instantaneous, accurate and complete information on the network congestion (we will say that the information is *perfect* in this ideal case), information which can then be optimally processed by all users in real time[5]. We feel that this requirements are very often not satisfied.

- Traffic patterns in large cities do change significantly over long periods. These changes are caused by several factors.
  - Transportation *demand* itself is far from being constant for such periods. Long term variations of the demand are caused, for example, by the evolution of transportation behaviour due to changing life-styles, increasing car-ownership, fluctuations of the costs of private and public transport (due in turn to petrol prices, insurances or public transport operators' marketing policies, for instance) The reader is referred to [43] for an interesting discussion of this and other related issues.
  - It is extremely unusual that the *structure of network* itself does not undergo major modifications in the horizon that would be necessary for "learning" the entire network: new tunnels, bridges, by-passes and one-way systems are indeed created on a nearly monthly basis in major cities. These creations are also often prepared by lasting perturbations to the existing surrounding network caused by the very building of the new infrastructures.
  - Even assuming the network structure and demand to be stable, the *network management* is in constant evolution, as signaling and traffic lights control are adapted.

  Assuming a network steady-state situation is therefore clearly at best questionable in our framework.

- One of the effects of advanced driver information systems is to produce substantial change in traffic patterns over short periods. Because their redistribute information related to the traffic situation relatively fast, they can (and should) induce re-routing of users just before or even during their trip. These short-term variations of network load are then superposed on top of the slower evolution just discussed, further blurring the user's perception of a network-wide steady-state.

- Complete information on the network status is very often not available to users. Even users connected to specialized dispatching centres only receive selected informations.

[5]It is interesting to note here that "quasi-equilibrium" situations may be reached where the information of users is not perfect: one might indeed think of an equilibrium situation where each user optimize his/her journey time over the restricted set of routes within his incomplete and/or inaccurate knowledge of the network. This of course requires that the route actually used is included in this knowledge, but this is not a severe restriction. Despite clear similarities, these situations however *differ* from the classical equilibrium case, which means that the equilibrium based computational techniques must be modified to handle them.

- The only information available is most of the time inaccurate, outdated or purely qualitative, making precise comparisons between contending routes impossible or at least very approximative.
- It is not clear at all that users, even provided with perfect information, will actually choose to use it, and then do so in an optimal way.

It is important to observe that the notion of equilibrium is further based on the assumptions that all network users are "identical" (using the terminology of [39]) in that they have a *common criteria* to optimize their mode choices and routes. However, recognition of variations in user behaviour is highly desirable for realistic modelling of the driver information systems impact. We have indicated above that these variations are often cared for by the introduction of a stochastic part in the utility functions, leading to what is called *stochastic user equilibrium* (SUE) techniques. Although a clear improvement on deterministic equilibrium, SUE is still subject to most of the other criticisms, including that mentioned above on the validity of random utility for modelling human behaviour and on the lack of a truly dynamic nature.

At this point we have provided arguments to show that equilibrium is not well suited to modelling evolving processes, especially fast ones. On the other hand, one must also admit that commuters in a network usually do not re-examine their transportation strategy every single morning. Hence our conclusions must be somewhat qualified.

We have seen that equilibrium models might be inadequate for three main reasons: they assume stability of network characteristics, rely on perfect information and assume uniform user behaviour. If it is true that those assumptions can be questioned in general, it is also true that their degree of inadequacy varies with the type of effects considered.

1. It still might be a reasonable approach for long term average traffic effects. This could be justified by the fact that not everything changes in the network, even for long periods. Some "experienced" knowledge is therefore not really outdated, even if it is incomplete, and can play a significant role in the basic unperturbed route decision process.
2. Some information systems deliver information of a very qualitative nature, whose variations in the long term are minimal. For example, congested spots are signaled daily in many radio bulletins: after some learning period, the knowledge that these spots are congested at particular times is incorporated into the broad common knowledge of all network users.
3. Finally, if individual variations in behaviour are obvious, one must still recognize certain major components in the decision rules used by classes of network users. This observation is the very base of the past success of equilibrium models. One might think along this line to use "parallel" equilibrium models for classes of users whose behavioural consistency is greater, either by simultaneously considering different group-dependent representations of the network, and/or different group-dependent cost functions, for instance.

One therefore needs to distinguish between the type of effects and impacts one wishes to analyze before accepting or dismissing equilibrium based models: they might not be suitable for modelling traffic in the presence of fast or unexpected problems or real time information systems, but could maybe play a role to describe general tendencies based on broad network knowledge, that would manifest in the restricted (and somewhat unlikely) case were no unexpected event or information perturbs the traffic situation.

We finally note that Papageorgiou [46] proposes an interesting approach that allows for transient effects in network load to take place, but still assumes perfect information and uniform user behaviour.

3.8.2 ATT and shortest paths techniques

As equilibrium assignment, the shortest paths technique assumes that all network user see the cost of their route in a uniform way. The variability inherent to human decision processes has very little room here, which can be judged unrealistic (see [28], for instance).

As an answer to this criticism, several stochastic variants of shortest path techniques have been proposed with the aim of re-introducing some variability while maintaining the assumption that the cost associated with a link is independent of traffic: the idea is to assign a random utility to "reasonable routes" or even to individuals links in the network.

The first of these technique, introduced by researchers like Dial around 1970 [20], consists in finding the set of reasonable routes between an origin and a destination. A route is said to be reasonable in this context when the users steadily increases its distance from the origin and decreases its distance to the destination when following the route. The shortest route is of course reasonable, but the set of reasonable routes is typically larger. The traffic is then spread over these routes, using a logit distribution function.

In the second method, one averages the flows obtained on a given link during successive application of the shortest path assignment method applied in a network whose link costs are drawn from a normal distribution with fixed mean. This "successive average" method correspond to applying the probit technique to the route choice problem.

We briefly summarize the relations between assignment methods in Table 1.

After our discussion of the notion of stochastic equilibrium assignment, we briefly note that the arguments presented in this framework unfortunately also apply to the simpler technique using randomized shortest path computations. In particular, they do not provide a natural way to consider route choice in a dynamic manner, a requirement which we saw is important in the framework of ATT applications. We note that [45] provides a dynamic generalization of the pure shortest path approach, but does not allow within trip rerouting.

4 Further limits of traffic modelling as an assessment tool

In this section, we briefly discuss some other limitations that should be kept in mind when using transportation models as an assessment tools for ATT, or for more general situations.

The first consideration is that a model can only really be used if it has been validated in real applications, and when its behaviour has been shown to be robust enough to allow reasonably safe

	constant costs	traffic dependent costs
deterministic approach	all-or-nothing	equilibrium (UE)
stochastic approach	reasonable paths or averaging	stochastic equilibrium (SUE)

Table 1: Relations between some traffic assignment techniques

extrapolation. While this is often the case for classical models which are ATT insensitive, this is much more rarely the case for newer and more advanced models (as can of course be expected). This remark is especially true for dynamic models, where validation is a much harder task. Hence the necessity of pilot experiments and associated model appraisal. Such pilot experiments have been conducted in the framework of DRIVE II and are discussed by other speakers in the course.

Correlative to the validation question is that of data collection. The more complex the model, the more data it typically requires, hence the more difficult to apply it with its full predictive power. This limitation has to be recognized, but does not necessarily prevent a cognitive use of the models as a tool to gain intuition and understanding of a practical situation.

In relation with the two above points, one could mention the conclusion drawn by Julien and Morellet in [32]: "Finally, the real debate on the choice of the type of modelling should not deal with what these or those models can do theoretically, but rather with the more or less important hazards to which the calibration of these models can be subjected". Although the author feels that this conclusion is probably too severe, it also has its part of truth...

A final and insidious limitation of the modelling methodology one finds in practical applications is the fact that, often, limited funding for the transportation study results in a partial modelling, where important aspects of the methodology (as public transport modal choices) are grossly simplified or neglected altogether. The model users obtain of course what they are paying for, but a too restrictive financial appraisal of the modelling phase might, and sometimes does, lead to substantial qualitative errors in the conclusions!

5 A partial review of some available modelling tools

It is of course very difficult to present an overview of the modelling packages that are available today. That attempted in this section does not pretend to be exhaustive, but rather follows the author's experience. For instance, it does not cover either models whose main purpose is traffic control (such as PRODYN, SCOOT, TRANSYT, to cite a few) or are restricted to motorways (as SIMAUT and METANET, for instance).

Before we start this overview, it is important to realize that a large gap exists between modelling research and modelling practice. And, according to experts participating in both activities, this gap is currently widening (see [8]). In particular, the "state-of-the-art" models use a finer and finer disaggregation, while the practitioners (consultants, administrative decision makers) keep on using very classical, well known and rather aggregate modelling techniques. For instance, it is reported that the standard four stages models as described above are the most widely used in transportation demand planning within the United Kingdom [5], while very advanced disaggregate models are also actively developed in the same country (see [31] for several examples). This trend is predicted to continue for a few years at least.

It therefore seems natural to first focus our model overview on the packages that are broadly used among practitioners. This is the object of this section.

5.1 SATURN

SATURN (for Simulation and Assignment of Traffic in Urban Road Networks) is a favourite model among specialized consultancy firms, and was developped by Dirk van Vliet and colleagues at the University of Leeds (UK) [51]. As its title implies, it covers the assignment phase (stage 4) of the classical four stages model, but also provides tools for improving an a priori known O/D matrix (stage 2). It is designed to cover moderately large network (of the order of 150 intersections). Its very interesting feature is a detailed dynamic simulation of the traffic flows at intersections.

SATURN provides both UE and SUE assignment methodologies (with a detailed simulation of intesections delays) and also allows the insertion of busses in the car flow. Its interface with the user is, up to now, rather primitive, data being exchanged via specialized files with rather rigid format. We are however aware that a better graphical interface is being developed.

From the point of view of ATT, SATURN is unfortunately not directly relevant. The main drawbacks are, from this point of view, its lack of behavioural differentiation (flows are used to represent traffic), its intrinsic static (steady state) character (traffic flows are assumed to be constant for periods of at least 30 minutes) and its lack of mechanism for modelling information exchange between different network parts.

SATCHMO, a public transportation model associated with SATURN, has just been announced.

5.2 TRIPS

Another very widely distributed modelling package is TRIPS. This package also covers traffic assignment (stage 4) and provide facilities to estimate an apriori O/D matrix (by a gravitational method) and to improve it to fit known traffic counts. The assigments procedures use methods somewhat older than those in SATURN (shortest path and stochastic variants, UE). TRIPS also provides functionalities to handle public transport, but they are often criticized as being very hard to calibrate. One of its major assets in its excellent user interface.

A recent extension to TRIPS [38] parametrizes the model's relevant quantities with time, using so called “flow profiles”. However, it is recommended to use time-slice of at least one hour, which makes it roughly equivalent to a sequence of successive static models. At variance with SATURN, no dynamic intersection simulation is provided.

Because of these restrictive features, TRIPS is not very suitable for the ATT context. It indeed lacks true dynamism, sensitivity to differences in behaviour and to information distribution .

5.3 EMME2

The Canadian model EMME2 [4] also provides traffic assignment (stage 4), given an O/D matrix. Its is based, at the large macroscopic level, on traffic equilibrium, and relies on rather detailed microsimulation for smaller delimited portions of the network. Its graphical interface is satisfactory and the package provides the user with an adapted modelling language to enable more specific applications.

The city-wide operation of EMME2 is adapted to the ATT context in that it is a static equilibrium model. However, the microscopic simulator provides facilities to analyze some traffic control strategies, a very useful feature in our framework. The micro-simulation can also help to refine the link cost functions for the more global equilibrium situation.

EMME2 is well known by consultants for its adaptability, and provides one of the best options for analyzing car and public transport traffic together.

5.4 CONTRAM

CONTRAM ([34], [35]) is a dynamic assignment model for moderately large networks, given a time-sliced O/D matrix. It has been developed by the British Transporation and Road Research Laboratory (TRRL). It assigns route to packets of typically about 10 vehicles in the network by an iterated shortest paths calculation. CONTRAM provides a detailed simulation of intersection, queues and trafic light cycles. It also handles, as SATURN and TRIPS, three different kinds classes of vehicles and the possibility of fixed routes, hence providing some support to realistic modelling of the impact of busses on urban car traffic.

In the context of ATT, CONTRAM still presents the problem to be based a rather long time-slices (of the order of 15 minutes) and also assumes that user possesses perfect information on the network, which is assumed to reach a steady-state.

5.4.1 A comment on demand modelling

As can be noted from the above descriptions, estimating the transportation demand and, to a lesser extent, the modal choice, are questions not well supported by the most popular packages. Many rely indeed on some a priori known O/D marix which is then improved with the help of traffic counts. But the initial building of the first matrix (i.e. stage 1 and most of stage 2) is left very much to the user's inspiration.

6 Behavioural models: a direction for the future

After discussing framework, methods, limitations and availability of transportation models, we now outline what is the direction in which these tools seem to evolve, albeit slowly: truly behavioural models. These models are largely new packages whose development is still ongoing, and whose validation, at variance with the more classical ones, is not yet fully completed.

6.1 A few desirable model features in the ATT framework

Based on the discussions of the previous sections, it seems natural to search for directions and features which would make modelling in ATT context a viable assessment tool. Besides a number of such features already mentioned as

- explicit dynamical structure,
- behaviourally coherent traffic metaphor, and
- explicit modelling of the information network,

It might be worthwhile to discuss two further concepts that have been considered in the PACSIM model, to be outlined below.

6.1.1 A flexible and standardized behavioural mechanism

We have discussed how much sensible behavioural modelling is important in our framework. But a first and important problem immediately arises: relevant behavioural theories are still very much in the development stage, and certainly no general agreement has been reached as to which theories do best represent the behaviour of network users in what are potentially extremely diverse situations. Therefore, there seems to be little point in incorporating one particular behavioural theory in a traffic model, and exclude other promising approaches at the same time. New models should therefore provide the capacity to use a particular *behavioural theory as a model parameter*. Furthermore, as several theories are likely to be of interest, the specific way in which models handle behavioural theories must be extremely flexible and adaptable.

Several possibilities exist to fulfill these goals, ranging from parametrized cost functions (probably too restrictive) to complete representation of the theory using knowledge representations systems, as developed in the area of artificial intelligence. A particular choice is indicated in the next section in the limited context of route planning for traffic assignment.

The author believes it is important that efforts are made towards a coherent way to represent behavioural theories, hopefully leading to some *standardization* in this research area. Furthermore, the representation format should allow simple and error free transmission to other researchers and, in the best case, be directly exploitable as such by modelling programs. These requirements are indeed crucial for these theories to play a really practical role in modelling methodology, even if they now appear to be somewhat idealistic. We refer the reader to [41] for a discussion of a potentially useful approach.

6.1.2 A flexible network representation

A very flexible transportation network representation is also extremely desirable in our "behaviourally driven" methodology. Indeed, users can only decide on route and mode choice within the framework of their knowledge of the network. But one knows from the research in mental maps (see [21]) that this knowledge might present relatively severe distortions from reality. It also typically involves some level of network abstraction (only major links are perceived in distant or unfamiliar districts).

The behavioural route or mode decision can then be broadly split into two phases, both driven by behavioural theory: the user knowledge is asserted first, followed by some choice between the decisions that are possible in the restricted context of this knowledge.

It is therefore crucial that the network representation can easily be adapted and modified (according to a given behavioural theory) to better reflect the user's perception of the network. This modification will thus typically involve network abstraction and distortions of various types (link suppression, cost modification, preferred routes, ...). This requirement in turn imposes the use of specialized and flexible data structures.

6.2 PACSIM: an approach to behaviourally driven route choice and assignment

After reviewing some desirable features for new traffic models, we devote this section to a brief survey of some of the modelling choices that were made for the traffic model PACSIM in compliance with our above comments.

The work on this model has started in the framework of the IMAURO project within the EEC DRIVE program. The purpose of the model is to provide an RTI/ATT sensitive traffic modelling technique which could then be used to assert the qualitative effects of advanced driver information systems.

It is not our purpose here to describe the PACSIM model in detail, which is done in [18]. We instead concentrate on some of the modelling concepts used in this approach, with the aim to show how their definition has taken the arguments developed above into account.

6.2.1 Modelling the information network

We stated above that an explicit model of the information flow from the network to its users is desirable. In accordance with this suggestion, PACSIM contains such an explicit model. This model takes itself the form of a network, that features four main type of nodes: detectors, traffic information centres, traffic control centres and information sockets. We now briefly review their function and relations.

The detectors' purpose is to monitor the status of the network and to detect any traffic event therein. For increased realism, detectors have individually adjustable sensibilities[6], reliabilities, reaction delay and can be positioned anywhere on the network.

[6] That is the type of effect and/or event that they are able to detect (as congestion, accident, unfavourable weather conditions, ...).

Traffic information centres are nodes at which information on detected traffic events is centralized, screened according to parametrizable selection criteria, and then distributed to other traffic information centres, traffic control centres and information sockets.

Informations sockets model the various devices that are used to transmit information to network users (radio beacons and dedicated broadcastings, variable message signs, ...). They come in different types, varying according to technology and capacity, but also in the amount of information they can store (memory), time during which information is considered up-to-date (persistence) and ways in which priority conflicts between different messages are handled. They can also be situated anywhere on the network.

Traffic control centres represent instances that decide on traffic management, control and optimization strategies, as opposed to mere information distribution. For instance, they can maintain sets of recommended routes for route guidance applications and fleet management. Recommendations and informations implementing these strategies are also distributed to network users via information sockets.

These four classes of "information network nodes" are themselves linked by communication lines whose reliability and delay can be parametrized. Furthermore, the complete information network configuration (topology and node/link parameters) can be dynamically updated during the design period.

Recalling the suggested features of the information network discussed in Section 2.1.4, we see that the particular choices made in PACSIM are indeed reasonably coherent with our previous propositions.

Again, the author realizes that the description of the information network in PACSIM might not capture every possible variant of information processing that can be thought of in the context of advanced driver information systems. However, the present definition already covers a fairly wide ground. It is also easily expandable if new technologies have to be modelled, that cause or result in new information flow patterns.

6.2.2 Time handling

An explicit time dimension was also recommended above for the type of models we have in mind. PACSIM is thus a dynamic model: it is event driven in the sense discussed in Section 2.2.1. Events in the model are of two distinct natures. The first type of event is associated with event happening in the transportation network itself, as occurrence of congestion, traffic light changes, arrival of cars at intersection, road works or traffic accidents. The second type of event is associated to the transfer of information throughout the information network: detectors actuation and information transmission across the communication network (including the specification of delays and information life-time).

6.2.3 Network abstraction

Our previous discussion also recommends the availability of network abstraction mechanisms, in order to reflect the different perceptions of a specific network location by users that are close to it and by users that are further away: close locations are perceived with more disaggregate detail while distant ones are perceived in a more aggregated fashion (where only main districts and major links matter).

According to this view, PACSIM uses a three levels hierarchical network representation. The three levels correspond to the *main* network, the *strategic* network and the *detailed* network, from the more aggregate to the more disaggregate. Explicit mappings are defined that assign a node of a disaggregate level to a node of the aggregate level just above, assuring operational coherency to the overall structure. This "multi-level" representation of the network of course plays an important role in the definition of the users' knowledge of the network (see Section 6.2.5), but also in other parts of the model, as described in the next section.

6.2.4 Packets

The traffic metaphor used in PACSIM is *based* on behavioural coherency, therefore complying with the above arguments closely. PACSIM indeed considers traffic as composed of the movements of *packets*, which are groups of vehicle moving on the network with a coherent behaviour. They are created at network locations corresponding to origins of transportation demand at a given time, travel through the network and disappear at their destination. The behavioural coherency of the packets is ensured by the requirement that vehicles belonging to the same packet must have in common

1. their capacity to access driver information systems and the traffic knowledge provided through this channel,
2. their mode of transportation,
3. their trip purpose,
4. their destination in the network.

Although the author realizes that these characteristics might not be sufficient to describe behaviourally coherent classes of users, their specification however constitutes a first attempt in this direction.

Given this definition of the packets, one is quickly faced with the problem that the number of packets present in the network at any moment of the design period might be very large, potentially causing a serious efficiency deterioration, therefore making a behaviourally coherent approach to traffic description questionable. PACSIM proposes one possible approach to control this number to some extent, which is based on the notion of abstracted destination. Given the multi-level representation of the network, the final destination of a trip (at the detailed level) can be abstracted at higher (more aggregate) levels when the trip making packet is still far from its destination. This technique ensures that, at some distance, all destinations within the same district, say, will be aggregated into the same abstracted destination[7]. As packets later approach

[7] Specialized data structures are again needed here: the present implementation uses a tree oriented description of the destination aggregation process, called the *destination tree*.

their aggregate destination, this destination is split into its more detailed components, and the packets are themselves split accordingly. Thus we see that packets not only move in the network, but also merge and split in a totally dynamic way. Packets at the beginning of their trips are merged with other packets sharing a common abstracted destination, while they can be merged with other packets approaching their true disaggregate destination at the other end of the trip, when the disaggregate destination itself is approached by several packets from possibly different origins.

6.2.5 The behavioural mechanism

The behavioural mechanism that drives PACSIM is organized along two complementary stages, broadly corresponding to the two phases for behavioural decisions outlined above. The user perception of the network is first established, and a routing decision is then taken within that restricted framework. We now briefly describes these two stages.

The network as perceived by the user (the packet), the *perceived network*, is first defined using the multi-level network representation, according to the following steps.

1. A perceived network is built, that contains a detailed view of the links and nodes in the surroundings of *anchor points*, with the level of detail fading away with distance. These anchor points represent parts of the network that are well known to the user. At present, the origin and destination of the trip are chosen as anchor points. This procedure makes full use of the multi-level network structure.

2. Behavioural rules applicable to the packet then modify this restricted network, according to the packet characteristics and the information it has accumulated in past time-slices. This modification can be of two possible natures:

   - the topology of the perceived network may be modified,
   - the relative costs of the perceived network links may be modified.

   In the first case, links can, for example, be removed from the network because they do not belong to the packet's knowledge of the area; links can also be added or reversed as a consequence of dynamic traffic management decisions (as tidal flows) which are communicated to the packet's via driver information systems. The modification of costs allows to reflect behavioural preferences (heavy goods vehicle drivers tend to prefer major links, for instance) or again to take available broadcasted information on network congestion into account.

Once the packet's knowledge of the network is established, the behavioural rules are further applied to determine the *route structure* as a sequence of subroutes. These subroutes are finally computed and assigned to the packet. PACSIM actually uses shortest path computation in the modified network for this very last step. It also features a further option to randomly perturb the network cost to represent individual stochastic variations within a single behaviourally coherent class of network users (in a spirit close to that of stochastic assignment, as described in [17] and [49] for example).

We mentioned above that certain behavioural rules are applicable to packets. The set of all these rules constitutes a behavioural theory of route planning, which is used as input parameter by PACSIM. These rules are specified in a formal language that has been specifically designed for the purpose, and which is described elsewhere (see [41]). We only stress here that this representation technique has several decisive advantages: formulated in such a formal language, the behavioural theory can be perspicuous, unambiguous, concise and machine readable. We also note that rules expressed in this language cover variations in behaviour potentially influenced by a wide range of parameters as availability, quality, accuracy and timing of user's information, location in the network, trip purpose, transportation mode, time of the day, ...

The inherent behavioural route choice of PACSIM imply that a mechanism is provided for the drivers to memorize some past informations for later use. However, routing information cannot be based on this information only, as has been already observed: in particular, a packet would not be able to start its route in the network because it is just, at this stage, beginning to accumulate information. Some degree of anticipation based on experience is thus necessary. PACSIM provides a mechanism (called the *background network*) to build up such long term dynamic experience of the network, which we saw was admissible in a limited sense. This provides the requested anticipation of traffic effects in the route planning decision process, at the condition that the knowledge of the anticipated effects can be acquired with experience. This long term experience is common to all drivers in the present version of PACSIM, but it might be envisaged to disaggregate it further to better reflect the differences in network experience between different users.

As we noted above, it is not clear what behavioural theory will actually prove to be most useful. Hence, the theory that is presently used by PACSIM (see [18]) is not aiming at completeness or even correctness: the best that can be hoped for is that something of the *character* of driver response to advanced information systems has been elucidated. One of its purposes is to allow the testing of the behavioural theory representation, particularly with regard to flexibility and efficiency. Better theories emerging from ongoing research projects within the DRIVE program and elsewhere can then be used and tested later without further modification to PACSIM.

6.2.6 Experience with PACSIM

The PACSIM model has been applied to simulate the morning peak hour in the city of Namur, and to optimize the localisation of variable message signs whose purpose is to warn users about traffic incidents in critical streets of the city.

This application has provided satisfactory results, at least on the cognitive level mentioned above, but we are still far from a full validation of the model. This validation is indeed a very difficult and lengthy task. Healthy scepticism should therefore by applied to our achievements. The author however feels that most of the idea presented in the framework of the PACSIM model are worth some discussion within the context of ATT sensitive models.

6.3 Some other new ATT sensitive models

6.3.1 SITRA-B+

We start our review of the newer models with some comments on a microscopic model, SITRA-B+, developed at CERT in Toulouse. This model has been designed with two main topics in mind: traffic light control stategies and vehicle guidance. It dynamically simulates the movement of each vehicle individually on a relatively small network, including detailed descriptions of intersections, parking (both on the road or dedicated). This simulation uses a car following model, that is a technique which determines the speed and acceleration of a vehicle based on its distance to the preceding vehicle on the same lane. The demand can be specified by time-sliced O/D matrices, by a dynamic description of flow at the network entry points or by specifying specific routes and timings, as is requested for bus lines.

It also contains some modelling of the information network introduced above, namely localized traffic measurement devices and radio beacons for information re-distribution to users.

SITRA-B+ has a flexible data interface, which allows its use in conjunction with more complex system, as the ASTERIX system developed in the context of the EC DRIVE project[8].

6.3.2 Extensions to SATURN and CONTRAM

Both SATURN and CONTRAM have been adapted by university researchers to take certain aspects of ATT into account. In particular, efforts have been made to remove from the original models the assumption of user perfect information that we already criticized above. For instance, a version of CONTRAM named ROGUS [30] incorporates random utilities and also offers facilities to simulate route guidance of the basis of an "historical" view of the network (very similar to that of background information within PACSIM) and the actual link cost as reported by guided vehicles.

Another extension exists [11] that divide the users in different classes (using the interesting notion of "familiarity" with the network) that obey to different route choices rules. This is again very much in the spirit of the behavioural rule approach outlined above, except that the rules seem to be hardcoded in the model.

These model extensions have been used by the research teams who produced them in some ATT scenarii evaluation, mostly in the context of the DRIVE program.

6.3.3 ATT sensitive models in the US

We cannot close this section without mentioning the active research being carried out in north America on dynamic ATT/ATIS sensitive models. Examples are the MIT Dynamic Traffic Model, DYNASMART or INTEGRATION. Most of them have abandonned the idea of steady state conditions in the transportaion network and provide, by different means, some sensitivity to the disaggregate user's behaviour, including en-route reassignment.

[8] ASTERIX also interfaces with the SATURN and CONTRAM models.

As the models mentioned in the previous paragraph, these models have been used mostly as research tools.

6.3.4 EUROTOPP

We conclude this section on new modelling ideas by a mention of the EUROTOPP model [3]. At variance with the preceding cases, this model does not deal mainly with traffic assignment, but instead focusses on dynamic transportation demand. It is based on the notion of activity chains and has been built to be ATT sensitive. Although very much in development, the ideas presented in this research effort appear as one of the very few methodologically sound approaches to this crucial problem.

7 Conclusion

It is the purpose of much of the contemporary research in transportation modelling to overcome the present limitations and restrictive assumptions in today's models, both from the methodological and practical points of view. Current trends consider fully dynamic models where a parametrizable user behavioural theory is used as an alternative to the notion of network wide equilibrium. Traffic modelling is also more challenging as new techniques are developed to help users and network managers to take adequate decisions in increasingly complex situations: radio-broadcasting of traffic messages, the use of variable message signs to suggest routes, interactive route guidance, automatic incident detection, road pricing and other demand management strategies clearly introduce new decision variables. The present paper has outlined major approaches in this area and their limits.

The relevance of transportation modelling in many polical and economical evaluations exercises must be considered with care, avoiding both excessive enthousiasm and premature dismissal. Models' purely predictive capacities can of course only be of real use in the measure where they have undergone careful calibration and validation. Unfortunately, this is not yet the case with many of the more advanced ATT sensitive approaches, which prompts for continued work in this direction. However, advanced modelling techniques can already play a significant role in their present stage of development, that of powerful cognitive tools. They are indeed irreplaceable in their ability to help building a deeper understanding of a situation of growing complexity, and thus truly contribute to establish coherent policies.

Advanced transportation modelling therefore remains important at the level of the transportation system managers' (and, to some degree, everyone's) task and responsability to appreciate the problems, methodologies and challenges in a domain where research, technology and everyday life are inextricably intertwined.

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