“Too young to die”. Deprivation measures combining poverty and premature mortality*

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Abstract

Most measures of deprivation concentrate on deprivation among the living population and, thus, ignore premature mortality. This omission leads to a severe bias in the evaluation of deprivation. We propose two different measures that combine information on poverty and premature mortality of a population. These measures are consistent and satisfy a number of desirable properties unmet by all other measures combining early mortality and poverty. Moreover, one measure is readily computable with available data and easily interpretable. We show that omitting premature mortality leads to an underestimation of total deprivation in 2015 of at least 36% at the world level.

JEL: D63, I32, O15.

Keyworks: Deprivation Measurement, Premature Mortality, Composite Indices.

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1 Introduction

"No winning words about death to me, shining Odysseus!
By god, I'd rather slave on earth for an other man-
some dirt-poor tenant farmer who scrapes to keep alive-
than rule down here over all the breathless dead."
Achilles' ghost to Odysseus, Homer, Odyssey.

Consider the evolution of Botswana at the end of the last century. In 1990, life expectancy in Botswana was 63.6 years while 33.6 % of its population was considered as extremely poor. In 2000, life expectancy was 45.6 years, while the proportion of extremely poor people had dropped to 29.5%.[1] Over a decade in Botswana, extreme (income) poverty decreased, but people also live a shorter life. The question we raise in this paper is how to evaluate, in a simple, meaningful and unambiguous manner, the evolution of total deprivation in Botswana between 1990 and 2000.

Deprivation is a multidimensional phenomenon (Alkire and Foster, 2011). The dimensions typically considered, such as income, education or health, only affect individuals when they are alive. In this paper, we consider instead *premature* mortality as an important source of deprivation (Sen, 1998; Deaton, 2013). Of course dying is not per se a form of deprivation: everyone is mortal and being deprived means falling short of a minimal standard in a welfare-relevant resource. However, an individual dying *too young* is deprived in the sense that she will not live a number of years considered as minimally acceptable. Under this approach, the resource of interest is the number of years spent alive, that is the lifespan.[2]

We propose two measures of total deprivation that explicitly take lifespan deprivation into account. The measures proposed so far in the literature are unsatisfactory either because, as most poverty indices, they simply ignore lifespan deprivation or because, as most composite indices, they account for it in an questionable way. More precisely, simple composite indices are not "consistent": they do not hold constant the trade-off between alive deprivation and lifespan deprivation.

To illustrate this point, consider the example given in Table 1, which compares three societies. In all societies, two individuals are born every year and no individual lives for more than two years. In society A, the two newborns are non-deprived and the two 1-year-old are (income) poor. As we assume the age threshold defining lifespan deprivation to be 2 years, no individual is lifespan deprived in society A. Society B is identical to society A except for the status of a 1-year-old individual: she is prematurely dead instead of being poor. Similarly, society C is identical to society B except for a 1-year-old individual who is prematurely dead instead of poor.

Income poverty is measured by the head-count ratio (HC), i.e. the fraction of alive individuals who are poor, which is 0.5, 0.33 and 0 in society A, B and C respectively. In this simple example, we can measure lifespan deprivation by the fraction of individuals born in the last 2 years who are already dead (LD). This fraction is equal to 0, 0.25 and 0.5, in society A, B and C respectively. A typical composite

[1] We present our databases below.

[2] This way of accounting for premature mortality is different from the missing poor approach followed by Lefebvre, Pestieau and Ponthiere (2013) and from the missing women approach (Anderson and Ray, 2010), where individuals dying in excess to a death rate are considered missing (see Section 4). We take an absolute deprivation approach to mortality, while the missing poor and missing women approaches take a counterfactual approach based on reference mortality rates.

Table 1: Composite indices are not consistent.

	0 year old	1 year old	HC	LD	$P_{0.5}$
Society A	Non-Poor, Non-Poor	Poor, Poor	0.5	0	0.25
Society B	Non-Poor, Non-Poor	Poor, Dead	0.33	0.25	0.29
Society C	Non-Poor, Non-Poor	Dead, Dead	0	0.5	0.25

The age threshold defining lifespan deprivation is 2 years

index of total deprivation simply aggregates the two dimensions by weighing them:

$$P_w = wHC + (1-w)LD$$

where $w \in [0,1]$ is the weight parameter w. Assuming $w = 0.5$, total deprivation as measured by $P_{0.5}$ is *smaller* in society A than in society B but *larger* in society B than in society C. Yet, comparing society B to A, or C to B, the only difference between those societies is that a single individual changed status, from being poor to being dead. We call these judgments "inconsistent" as they do not satisfy a basic separability property. They arbitrarily imply that being poor is worse than prematurely dead in some situations but better in other situations. This inconsistency arises because the two measures that compose the index, HC and LD, are based on different reference populations: the living population for HC, and the "total" population for LD. We discuss these inconsistencies in more details in Section 4.1 and show that they affect commonly used indices such as the Human Poverty Index (Watkins, 2006).

The two indices of total deprivation we propose, Inherited Deprivation (ID) and Generated Deprivation (GD), explicitly combine alive deprivation and lifespan deprivation in a consistent and straightforward manner. These indices also satisfy a number of desirable properties unmet by all other measures combining alive deprivation and premature mortality. Our theoretical approach provides the foundations for a particular aggregation of alive deprivation and lifespan deprivation based on time units. More precisely, our indices aggregate person-years in alive deprivation (PYADs) with person-years prematurely lost (PYPLs), given an age threshold below which dying is considered as premature. These indices therefore measure the incidence, and not the intensity, of alive and lifespan deprivation.

ID is based on past mortality, and records the number of individuals who died prematurely in the past but should have been alive today. GD is based on current mortality, as measured by the number of years prematurely lost by individuals dying in the current period. Compared to ID, GD measures how much deprivation has been generated in the year considered, and better corresponds to a flow measure of deprivation. This difference also makes GD more sensitive to contemporaneous changes in the society. Moreover, GD is in practice easier to compute than ID, given the type of data available. As we discuss later, the construction of ID requires a history of mortality rates while GD only relies on current mortality rates.

Three main lessons can be drawn from this exercise. First, when aggregating different dimensions of deprivation, lifespan deprivation should be treated separately. The fundamental reason lies in the *exclusive* nature of this dimension: individuals, once dead, cannot be considered as deprived along another dimension. This also implies that, to measure total deprivation, a lifespan deprivation component can be added to an alive deprivation component. Second, when measuring total deprivation in a given year, the lifespan deprivation component should be measured in *time units*, i.e. the number of years prematurely lost due to early death. Fundamentally, alive deprivation is also measured in time units since it records the number of (alive) individuals who are poor *in a given year*, which corresponds to the number of years spent in poverty by a population in a given year. Third, a familiar critique of composite indices is that they typically rely on arbitrary weights, which weakens their relevance when the dimensions considered vary in opposite directions. Our analysis instead provides a normative support for a lower bound on the relative weight of premature mortality, based on the idea that one year prematurely lost is at least as bad as one year spent in alive poverty.

Using data sets on income deprivation (PovCalNet) and on mortality (Global Burden of Disease), we show that, for the 1990-2015 period in the developing world, lifespan deprivation is not negligible as compared to income poverty. The omission of lifespan deprivation leads to an underestimation of global total deprivation of at least 27 to 36% during the whole period. In 2015, there were 705 million income poor individuals (PYADs) and premature mortality in the same year caused the loss of 402 millions person-years (PYPLs). Moreover, the relative importance of lifespan deprivation in total deprivation has been increasing over time: the omission of premature mortality from deprivation measures therefore leads to an increasing bias.

At the country level, important differences arise between alive deprivation and total deprivation, and the evolution of total deprivation sometimes contradicts the evolution of income poverty for several countries and periods. Thus, for 8% of the country-periods considered, total deprivation evolves in the opposite direction to income deprivation. Deprivation assessments ignoring premature mortality at the country level are therefore seriously biased, and may lead to flawed policy evaluations.

The remainder of the paper is organized as follows. We first present the two indices, and discuss some of their properties in Section 2. We also investigate their dynamic behavior. A complete characterization of the two indices is given in Section 3. We then compare the fundamental differences between our indices and the alternative approaches proposed so far in the literature in Section 4. An empirical description of the evolution of total deprivation at the world and at country level is presented in Section 5 and Section 6 concludes. All proofs are relegated to the Online Appendix.

2 Two families of total deprivation measures

2.1 Basic framework

In this section, we define our two measures of total deprivation, combining in a single index alive deprivation and lifespan deprivation. In period t, each individual i is characterized by a **bundle** $\mathbf{x}_i = (b_i, s_i)$, where $b_i \in \mathcal{Z}$ is her birth year with $b_i \leq t$ and s_i is a categorical variable capturing individual status in period t, which can be either alive and non-poor (NP), alive and poor (AP) or dead (D), i.e. $s_i \in \mathcal{S} = \{NP, AP, D\}$. In the following, we often refer to individuals whose status is AP as "poor". We consider here that births occur at the beginning while deaths occur at the end of a period.[3] As a result, an individual whose status in period t is D died before period t.[4]

Let $a_i = t - b_i$ be the age that individual i would have in period t given her birth year b_i. We define a lifespan threshold $\hat{a} \in \mathcal{N}$, below which a lifespan is normatively considered too short. This threshold, which does not depend on the lifespan distribution in the population, corresponds to an "absolute" approach of lifespan deprivation.[5] An individual "dies prematurely" if she dies before reaching the minimal lifespan. Formally, period t is "prematurely lost" by any individual i with $s_i = D$ and $a_i < \hat{a}$. A **distribution** $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_{n(\mathbf{x})})$ specifies the birth year and the status in period t of all $n(\mathbf{x})$ individuals. The set of distributions, denoted by $\mathcal{X}$, is formally defined in Section 3.

This framework extends the traditional approach used in poverty measurement in two ways: to all individuals is attached a birth year and some individuals may be dead. A total deprivation index ranks all distributions in the set $\mathcal{X}$ as a function of the deprivation that they contain. Formally, it is a function $P : \mathcal{X} \to \mathcal{R}_+$, where $P(\mathbf{x}) \geq P(\mathbf{x}')$ means that $\mathbf{x}$ has weakly more deprivation than $\mathbf{x}'$ and strictly more if $P(\mathbf{x}) > P(\mathbf{x}')$. (Age threshold $\hat{a}$ is assumed fixed.) Note also that the three status framework $\{NP, AP, D\}$ is intentionally restrictive in order to focus on the aggregation of lifespan deprivation with other forms of deprivation. Our results can easily be extended to richer structures where individual achievements while alive are measured in multiple dimensions.[6] These achievements could also be measured using continuous rather than categorical variables, thereby accounting for the intensity of deprivation, but we stick here to the incidence.

By construction, classical deprivation indices do not measure lifespan deprivation. Consider the following distribution in period t with three individuals

$$\mathbf{x} = ((young, NP), (young, D), (old, D)),$$

where a birth year which is at least $\hat{a}$ years distant from year t is noted as *old*, and *young* otherwise. Because she is young and dead, individual 2 has prematurely lost period t.

We contrast distribution $\mathbf{x}$ with two alternative distributions $\mathbf{x}'$ and $\mathbf{x}''$ in period t that are both obtained from $\mathbf{x}$ by changing the status of individual 2. In $\mathbf{x}'$, individual 2 is alive and non-poor, while in distribution $\mathbf{x}''$ individual 2 is alive and

[3] This assumption implies that lifespans have no decimals, and is made for expositional reasons. We could alternatively assume that all deaths take place at the beginning of the period, which is the assumption made in the empirical section for practical reasons.

[4] All newborns have age 0 during period t and some among these newborns may die at the end of period t. This implies that $b_i = t \Rightarrow s_i \neq D$.

[5] The introduction of an age-threshold is in line with the methodology used in the literature on multidimensional poverty measurement, which assumes dimension-specific thresholds in order to define dimension-specific deprivation status (Alkire and Foster, 2011; Pattanaik and Xu, 2018).

[6] In this richer framework, we would need to define dimension-specific deprivation thresholds, impose a series of classical axioms that would constrain how to aggregate the continuous achievements in these multiple dimensions, and ultimately obtain a classification of individuals into those who are multidimensionally deprived and those who are not multidimensionally deprived. The first category could be described by a continuous multidimensional poverty score, in the vein of Alkire and Foster (2011). In order to simplify the exposition, we directly assume this score to be zero or one.

poor, i.e.

$$\mathbf{x}' = ((young, NP), (young, NP), (old, D))$$
$$\mathbf{x}'' = ((young, NP), (young, AP), (old, D)).$$

These three distributions are compared in Table 2.

Table 2: Comparing distributions by changing the status of one individual

	(young, AP)	(young, NP)	(young, D)	(old, D)
Distribution $\mathbf{x}$	0	1	1	1
Distribution $\mathbf{x}'$	0	2	0	1
Distribution $\mathbf{x}''$	1	1	0	1

In these three distributions, no individual is alive and poor, except individual 2 in distribution $\mathbf{x}''$. As a result, the head-count ratios (HC) of distributions $\mathbf{x}$ ($HC = 0/1$) and $\mathbf{x}'$ ($HC = 0/2$) are identical and equal to zero, while the head-count ratio of $\mathbf{x}''$ is equal to $1/2$.[7] However, distribution $\mathbf{x}'$ is arguably better than distribution $\mathbf{x}$, since individual 2 is not prematurely dead in $\mathbf{x}'$. Moreover, it is not clear that distribution $\mathbf{x}''$ is worse than distribution $\mathbf{x}$: individual 2 is poor in $\mathbf{x}''$ but prematurely dead in $\mathbf{x}$. Whether distribution $\mathbf{x}$ is preferable to distribution $\mathbf{x}''$ is a judgment based on how one compares spending period t in poverty to prematurely losing period t. In our epigraph, for example, Achilles clearly states that spending a year in poverty is much preferable than spending a year in lifespan deprivation: Achilles would consider that distribution $\mathbf{x}$ is much worse than distribution $\mathbf{x}''$.

2.2 The inherited deprivation index

We first introduce an index based on mortality inherited from the past and refer to this index as the inherited deprivation (ID). Let $d(\mathbf{x})$ denote the number of *prematurely dead* individuals in distribution $\mathbf{x}$, which is the number of individuals i for whom $s_i = D$ and $\hat{a} > t - b_i$, $p(\mathbf{x})$ the number of individuals who are poor and $f(\mathbf{x})$ the number of alive and non-poor individuals. ID is defined as

$$ID_\gamma(\mathbf{x}) = \underbrace{\frac{p(\mathbf{x})}{f(\mathbf{x}) + p(\mathbf{x}) + d(\mathbf{x})}}_{alive\ deprivation} + \gamma \underbrace{\frac{d(\mathbf{x})}{f(\mathbf{x}) + p(\mathbf{x}) + d(\mathbf{x})}}_{lifespan\ deprivation}, \tag{1}$$

where $\gamma > 0$ is a parameter weighing the relative importance of alive deprivation and lifespan deprivation. An individual losing prematurely period t matters γ times as much as an individual spending period t in alive deprivation.

Index ID_γ has an alive deprivation component (poverty) and a lifespan deprivation component (premature mortality). The alive deprivation component records the number of persons who are poor in period t, and the lifespan deprivation component records the number of persons who were born less than $\hat{a}$ years before t but have already died. The denominator of both components is identical and equal to the reference population. This reference population includes all individuals, whether

[7] The comparison of distribution $\mathbf{x}''$ to distribution $\mathbf{x}$ is an example of the "mortality paradox": the reason why the HC of $\mathbf{x}''$ is higher than that of $\mathbf{x}$ is because the poor individual of distribution $\mathbf{x}''$ is dead in distribution $\mathbf{x}$. We discuss this question in more details in Section 4.

dead or alive, born less than $\hat{a}$ years before t, as well as all older individuals still alive in t.

Comparing distributions $\mathbf{x}$ and $\mathbf{x}'$ given above, the inherited deprivation index considers distribution $\mathbf{x}$ as unambiguously more deprived than distribution $\mathbf{x}'$. By contrast, classical deprivation indices, such as HC, are not able to capture a difference between these two distributions.[8] This is because these indices satisfy an Independence of Dead property, according to which the presence of an additional dead individual (all properties are formally defined in Section 3) leaves them unaffected. As a result, they ignore prematurely dead individuals.

By contrast, the inherited deprivation index captures premature mortality. A priori, a distribution contains all individuals that ever lived in a particular society. We impose a Weak Independence of Dead property, implying the index is not affected by the presence of an additional dead individual, if this individual is born at least $\hat{a}$ years before period t. This last property defines the relevant population in period t by excluding two types of individuals: those who died after reaching the age threshold and those who died below the age threshold but too far away in the past. Among the dead individuals, only those who died prematurely and whose birth year is less than $\hat{a}$ years before t are considered as part of the reference population.

When comparing distributions $\mathbf{x}$, $\mathbf{x}'$ and $\mathbf{x}''$ given above, ID focusses on old individuals who are alive and all young individuals, whether alive or not (as required by the Weak Independence of Dead property). In all three distributions, the reference population is composed of two individuals. As individual 2 is prematurely dead in distribution $\mathbf{x}$ whereas she is alive and non-poor in distribution $\mathbf{x}'$, $ID_\gamma(\mathbf{x}) > ID_\gamma(\mathbf{x}')$. In addition, as individual 2 is prematurely dead in distribution $\mathbf{x}$ whereas she is alive and poor in $\mathbf{x}''$, $ID_\gamma(\mathbf{x}) \geq ID_\gamma(\mathbf{x}'')$ when $\gamma \geq 1$. The larger premature mortality in $\mathbf{x}$ more than compensates for the larger alive deprivation in $\mathbf{x}''$, and ID contradicts HC.

The implementation of ID involves two important normative choices: (i) the choice of $\hat{a}$, the age threshold below which the death of an individual is considered as premature and contributes to total deprivation and (ii) the value of γ, the parameter weighing the relative importance of poverty and premature mortality. We believe that $\gamma \geq 1$ is a meaningful constraint, as one year "not lived" in a young age can be considered as at least as undesirable as one year spent in poverty.[9] A revealed preference argument supports $\gamma \geq 1$ given that committing suicide is an outside option (plausibly) available. Of course, some people do commit suicide. In particular, Bantjes et al. (2016) document that poverty is associated with mental illnesses leading to suicide. However, the constraint $\gamma \geq 1$ is relevant as long as the fraction of "young" individuals who prefer to be dead instead of poor is quantitatively negligible.

In Section 3, we show that ID is (completely) characterized by a small number of desirable properties. In particular, our characterization implies that alive and lifespan deprivation enter the index in an additive way, so that computing ID amounts to a very basic accounting exercise. The fundamental intuition underlying this addi-

[8] Prematurely dead individuals are not *intrinsically* valued by the HC. In practice, the HC is *instrumentally* affected by premature mortality when premature mortality is selective, i.e. when it affects poor individuals more than non-poor individuals. See Ravallion (2005) for an assessment of the contribution of both selective mortality and selective fertility to measures of poverty.

[9] Assuming $\gamma < 1$ would imply that a policy whose sole impact is to delay the premature death of a poor individual by one year increases total deprivation, an arguably dubious judgment.

tive separability is that an individual cannot simultaneously be "prematurely dead" and "poor": these two statuses are mutually exclusive, which allows us to sum the number of prematurely dead individuals and the number of individuals affected by alive deprivation. In contrast, non-exclusive dimensions of alive deprivation, such as income and health deprivation, would not necessarily be additively separable as the same individual can be simultaneously income and health deprived.

Relatedly, our definition of a distribution does not simultaneously contain information about an individual deprivation status and on her chances of survival. This particular assumption, which we discuss more carefully at the end of Section 4, is motivated by the near absence of comparable data sets that simultaneously contain at the individual level information about lifetime duration and deprivation status. Our measures are therefore indifferent to the joint distribution across individuals of periods spent in alive deprivation and periods prematurely lost.

2.3 The generated deprivation index

ID is an intuitive and straightforward manner to include premature mortality in deprivation measures. By definition, it measures current lifespan deprivation resulting from past mortality. This makes its empirical implementation difficult as its computation requires detailed information on mortality of each age cohort for all $\hat{a}$ years preceding t. Also, the impact of a mortality shock, whether permanent or temporary, takes decades to be fully accounted for. This implies that ID exhibits inertia which may be undesirable when used to evaluate the impact of contemporary public policies. For instance, today's ID for Rwanda's still accounts for children who died during the genocide of 1994: this is probably an accurate picture of total deprivation in Rwanda, but of little use to evaluate its current policies. The alternative index we propose, called the Generated Deprivation Index (GD) does not suffer from these limitations. It shares closely related properties and is based on the same intuition as ID. However, it is defined on current, instead of past, mortality rates.

Consider the population pyramid in period t, and let $n_a(\mathbf{x})$ be the number of *alive* individuals of age a in distribution $\mathbf{x}$, i.e. the number of individuals i for whom $a_i = a$ and $s_i \neq D$. Letting $d_a(\mathbf{x})$ be the number of *dead* individuals born a years before t in distribution $\mathbf{x}$, the total number of individuals born a years before t is then equal to $n_a(\mathbf{x}) + d_a(\mathbf{x})$. The age-specific mortality rate $\mu_a \in [0,1]$ denotes the fraction of alive individuals of age a dying at the end of period t: the number of a-year-old individuals dying at the end of period t is $n_a(\mathbf{x}) * \mu_a$. Letting $a^* \in \mathcal{N}$ stand for the maximal lifespan (which implies $\mu_{a^*} = 1$), the **vector of age-specific mortality rates** in period t is given by $\mu = (\mu_0, \ldots, \mu_{a^*})$. Vector μ summarizes mortality in period t, while distribution $\mathbf{x}$ summarizes alive deprivation in period t as well as mortality before period t.[10]

The generated deprivation index (GD) is defined as follows:

$$GD_\gamma(\mathbf{x}, \mu) = \underbrace{\frac{p(\mathbf{x})}{f(\mathbf{x}) + p(\mathbf{x}) + d^{GD}(\mathbf{x}, \mu)}}_{alive\ deprivation} + \gamma \underbrace{\frac{d^{GD}(\mathbf{x}, \mu)}{f(\mathbf{x}) + p(\mathbf{x}) + d^{GD}(\mathbf{x}, \mu)}}_{lifespan\ deprivation} \tag{2}$$

[10] Observe again that this framework is consistent with our data constraint. A pair $(\mathbf{x}, \mu)$ does not simultaneously contain information on an individual's deprivation and her chances of survival.

where d^{GD} measures the number of person-years prematurely lost due to deaths occurring in period t:

$$d^{GD}(\mathbf{x}, \mu) = \sum_{a=0}^{\hat{a}-1} n_a(\mathbf{x}) * \mu_a * (\hat{a} - (a+1)). \tag{3}$$

Like ID, GD is the sum of an alive deprivation component, recording the number of person-years in alive deprivation (PYADs), and a lifespan deprivation component. The lifespan deprivation component of GD differs from that of ID, as it records the number of person-years prematurely lost (PYPLs) generated by deaths occurring in period t. When an individual dies at age $a < \hat{a}$, she prematurely loses $\hat{a} - (a+1)$ periods of life. GD records these $\hat{a} - (a+1)$ PYPLs and assigns them to the year during which the death occurs. By contrast, ID records all the PYPLs in period t that were generated by deaths occurring before period t. The denominator of GD is analogous to that of ID, as it simply adds the number of alive individuals in period t to the number of PYPLs.

GD and ID are similar in many ways. In particular, a person-year lost due to a "mature" death does not enter the reference population, all PYPLs have the same weight and the weight γ of a PYPL relative to a year in alive deprivation is constant. Moreover, as discussed in Section 3, they are both *additively* decomposable. The main difference between the two is that GD relies on *current* mortality while ID relies on *past* mortality.

While, in general, *current* mortality is a priori unrelated to *past* mortality, the two coincide in *stationary populations*. In a stationary population, the number of newborns and the mortality vectors are constant over time, so that the population pyramid in period $t+1$ replicates the population pyramid in period t. Formally, the pair $(\mathbf{x}, \mu)$ is **stationary** if, for some $n^* \in \mathcal{N}$ and all $a \in \{0, \ldots, a^*\}$, we have:

- $n_a(\mathbf{x}) + d_a(\mathbf{x}) = n^* \in \mathcal{N}$ (constant natality),
- $n_{a+1}(\mathbf{x}) = n_a(\mathbf{x}) * (1 - \mu_a)$ (identical population pyramid in $t+1$).

In a stationary pair, the population pyramid is such that the size of each cohort can be obtained by applying to the preceding cohort the current mortality rate.[11] In such a case, past and current mortality coincide and the mortality vector μ does not convey any information that cannot be inferred from the population pyramid associated to distribution $\mathbf{x}$. As vector μ is redundant, a deprivation index can be computed from the distribution $\mathbf{x}$ only.

Our characterization of ID (see Section 3) shows that, when measuring deprivation on distribution $\mathbf{x}$ only, one should use ID. We therefore impose an ID Equivalence property, requiring that, for stationary pairs, total deprivation indices correspond to ID. This property implies that deprivation indices agree with ID on the *long-run* consequences of permanent changes in mortality or natality rates. GD satisfies this ID Equivalence property.

As formally proven in Section 3, the GD_γ index and ID are identical in *stationary populations* because d^{GD} coincides with d in that case. In stationary populations,

[11] Such population pyramids correspond to the ones prevailing in the long run if current mortality and natality rates remain constant over time (see for instance Preston, Heuveline and Guillot (2000)).

counting the number of individuals who prematurely miss period t due to past mortality is equivalent to counting the number of person-years lost due to premature mortality in period t. The fundamental intuition for this equivalence is illustrated in Figure 1. The left panel shows that d counts "vertically" the number of individuals who are younger than $\hat{a}$ years and died before period t. The right panel shows that d^{GD} counts "horizontally", for each age group below $\hat{a}$, the number of person-years prematurely lost by individuals in that age group who die in period t. When the mortality rates of the young correspond to the population pyramid, the two shaded areas coincide.

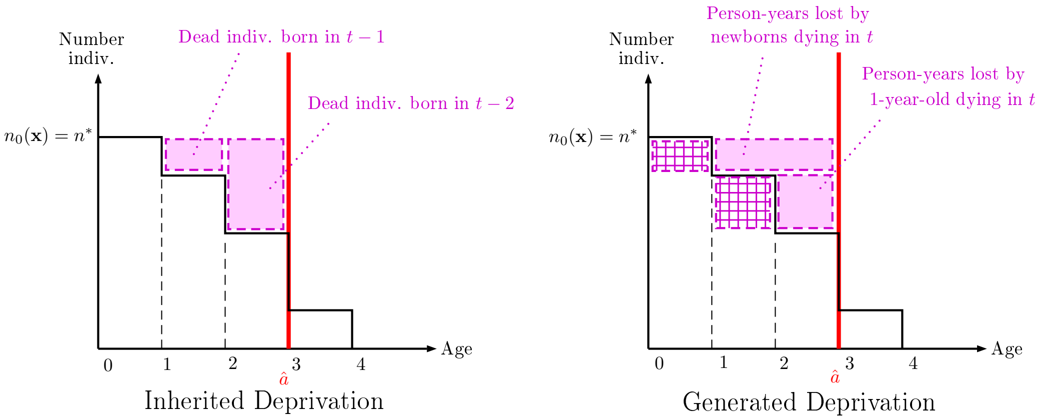


Figure 1: Left panel: The shaded area above the population pyramid represents $d(\mathbf{x})$. Right panel: The shaded area above the population pyramid represents $d^{GD}(\mathbf{x}, \mu)$ and the hatched areas represent the age-specific number of deaths in the period.

2.4 Dynamic behavior of the two indices

GD is equivalent to ID for stationary populations. Actual populations however are typically non-stationary. Permanent and transitory mortality shocks regularly affect population pyramids, which take decades to adjust to these shocks. In this section we compare the behavior of our two indices in non stationary populations and investigate their reactions to different types of mortality shocks.

2.4.1 Transitory mortality shocks

We first investigate responses to a transitory mortality shock in a simple example. We consider a population with a fixed natality $n_0(\mathbf{x}) = n^* = 1$ for all periods t. At each period, all alive individuals are non-poor, implying that $HC(\mathbf{x}) = 0$. For all $t \neq 0$, we assume a constant mortality vector $\mu = \mu^* = (0, 0, 1)$, so that each individual lives exactly three periods. Let us fix the normative parameters at 1 for γ and 3 for $\hat{a}$, so that an individual dies prematurely if she dies before her third period of life. Before period $t = 0$, the population is stationary, and the two indices are equal to zero since there is no poor and no premature deaths. Let us now consider a one period shock at period 0, such that all individuals die: $\mu^0 = (1, 1, 1)$. After the shock, mortality rates directly come back to their initial value and the population pyramid returns to its stationary state in period 3, after a (mechanical) transition in period 1 and 2 during which the newborns of period 1 and 2 grow up. This example is illustrated in Figure 2, where the reference populations are denoted by n^{ID} and n^{GD}, for ID and GD respectively.

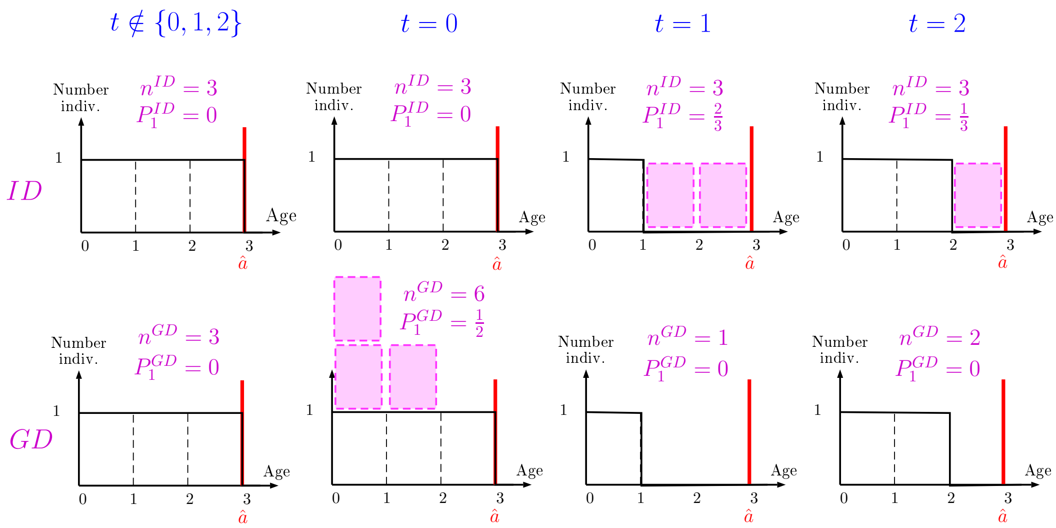


Figure 2: Response of ID and GD to the transitory mortality shock in $t^* = 0$. The person-years prematurely lost are shaded.

Consider first ID. In period 0, no premature deaths are recorded, since they all happen at the end of period 0. The number of person-years prematurely lost recorded by ID is equal to 2 in period 1, 1 in period 2 and 0 afterwards, as illustrated by the shaded areas in the first row of Figure 2. Given that one individual is born in every period and $\hat{a} = 3$, the relevant population is given by $n^{ID} = 3$ in all periods. Therefore, ID is equal to 2/3 in period 1 and 1/3 in period 2.

GD records the shock immediately in period 0. The newborn who dies in period 0 produces 2 PYPLs and the individual aged 1 in that period produces 1 PYPL. To compute GD in period 0, we consider a total of 6 person-years and GD is equal to 1/2 in period 0. Since the newborn in period 1 does not die in period 1 and is the only individual alive, GD records one PY with no deprivation and no PYPL. It is therefore equal to 0. Similarly, for period 2, there are 2 individuals alive, but no deprivation, and GD is again equal to 0.[12] Note that both ID and GD record the premature loss of three person-years. This is no coincidence, as we show in the Online Appendix. In a stationary population affected by transitory mortality shocks, GD and ID indices compute the same number of PYPLs, but distribute these PYPLs over different periods of time.

There are many instances of large transitory mortality shocks in history. For example, in 2010, Haiti was hit by a devastating earthquake, killing hundreds of thousands.[13] Figure 3 presents the evolution of the lifespan component of ID and GD with $\hat{a} = 5$.[14] Between 1995 and 2009, the lifespan components of ID and GD are quite similar. However, the person-years prematurely lost due to the earthquake are

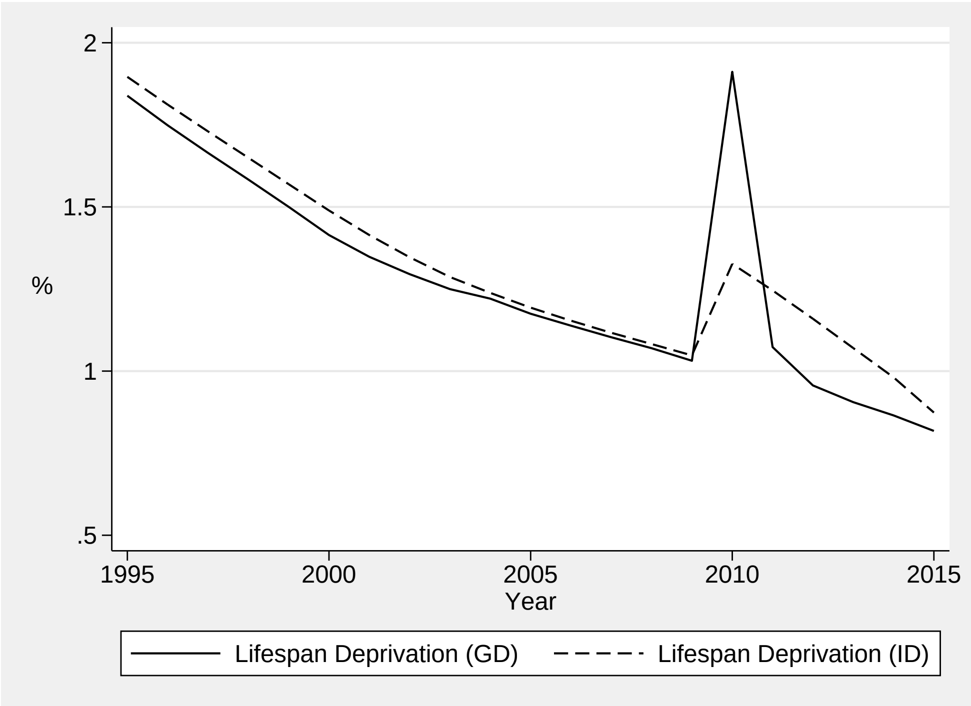
[12] Note that the fact that the index returns to its initial value after one period is a particularity of this simple example. If instead we had $n^* = 4$, $\mu^0 = (1/2, 1, 1)$ and $\mu^* = (0, 1/2, 1)$, the index would take longer to return to its stationary value.

[13] 315,000 according to the Global Burden of Disease, our main data source described in Section 5.1.

[14] We use this low threshold to illustrate the equivalence between ID and GD in the "long run". Given that our data ends in 2015 and the earthquake took place in 2010, $\hat{a} = 5$ is the highest threshold we could use for this example.

distributed differently. The GD approach attributes them all to 2010: there is a large spike in 2010, and a return to the long term trend right afterwards. The evolution of ID is different, with a smaller spike in 2010 but values that remain above the trend for the 4 subsequent years: it is only in 2015, i.e $\hat{a}$ years after the earthquake, that the ID's lifespan component returns to its long term trend.

Figure 3: Evolution of the lifespan components of ID and GD in Haiti ($\hat{a} = 5$)



Source: Global Burden of Disease, 2017. Authors' calculations.

2.4.2 Permanent mortality shocks

We now investigate the consequences of a permanent mortality shock on a stationary population. After a mortality shock, a transition phase sets in during which the population pyramid adjusts to the new mortality vector till the population reaches a new stationary equilibrium. This transition takes several decades and is particularly long for mortality shocks affecting young individuals. During this transition, the two indices are not equivalent.

To illustrate this point, we use simulations, assuming constant natality rates and no alive deprivation. The age threshold is 50 and the maximal age is 100. Before the shock, the population pyramid is consistent with a mortality vector such that, at each age before 100, the mortality rate is equal to 2%. Figure 4 illustrates the relative evolution of the two indices for three types of permanent shocks: (1) the mortality rates fall from 2 to 1% for all ages, (2) the mortality rate falls from 2 to 1% only at age 40 and (3) the mortality rate falls from 2 to 1% only at age 10.

The upper graph illustrates the consequences of the uniform mortality shock, the middle graph of the mortality shock at age 40 and the bottom graph of the mortality shock at age 10. The two indices evolve very differently over the transition period. In all scenarios, ID remains unaffected during the period of the shock, but

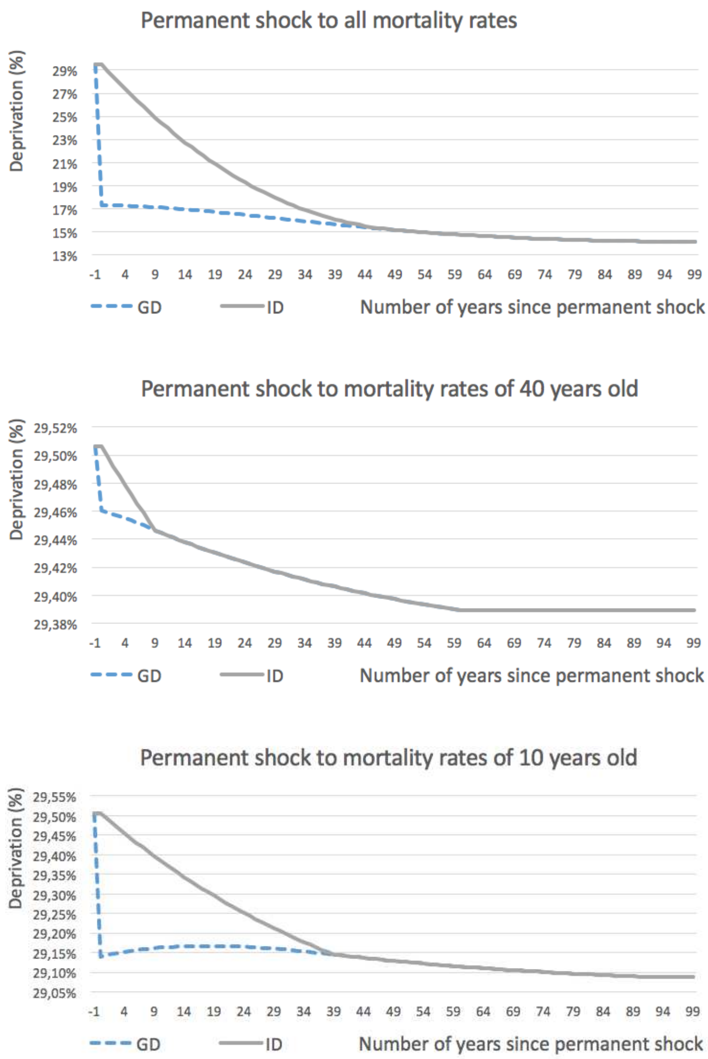


Figure 4: Simulation of permanent mortality shocks on a stationary population ($\hat{a} = 50$)

adjusts in a smooth monotonic way afterwards. GD jumps discretely in the period of the shock, and continues to slowly adjust to the induced changes in the population pyramid. In the long run, the two indices are equal. Note that when mortality falls, GD is systematically lower than ID until they converge again once the shock is fully accounted for.

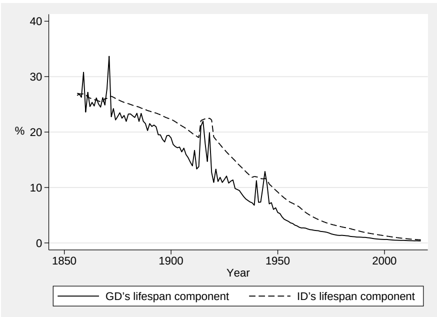
These simulations indicate that GD is more reactive than ID to a permanent mortality shock. Past natality and mortality still affect GD indirectly by shaping the current population pyramid on which it is defined. GD therefore partly reflects deaths that occurred in the past, even if the magnitude of this inertia is much smaller than that of ID. Moreover, the dynamics of premature mortality is determined by the interaction between the population pyramid and the mortality vector, and the relative size of young age cohorts in the current population pyramid may not evolve monotonically. This explains why the evolution of GD is not necessarily monotonic during the transition, as shown in the bottom graph of Figure 4. We provide in the Online Appendix another illustration of this property.

2.4.3 An historical application: the case of France

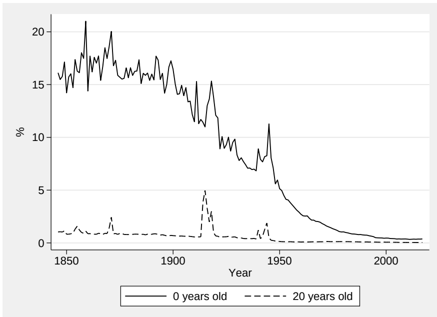
We now provide a comparison of our two indices for the case of France. More precisely, in the absence of comparable poverty measures throughout the period, we focus on the lifespan component of the two indices and compare their evolution over time. The data used comes from the Human Mortality Database (University of California, Berkeley , USA). The French age-specific mortality database begins in 1816 and is the longest series available in the Human Mortality Database. In order to discuss the period prior to 1870, we use a 40 years threshold (instead of 50 years used elsewhere in the paper).

The left panel of Figure 5 presents the evolution of the lifespan component of ID and GD for France between 1856 and 2010, while the right panel reports the mortality rates of individuals aged 0 and 20 years for the same period.

Figure 5: Comparing ID and GD: France



France: lifespan component of ID and GD, with $\hat{a} = 40$



France: evolution of mortality rates at ages 0 and 20

Prior to the Franco-Prussian war in 1870, the lifespan components of ID and GD are very similar, suggesting that the French population in those days was close to stationary. The casualties of the 1870 Franco-Prussian war lead to a large increase in the number of PYPLs, which is reflected by the spike in GD, which attributes them all to the year 1870. By contrast, ID exhibits a discrete jump which is much less pronounced but of a much longer duration. As discussed above, this is due to the fact that ID allocates the PYPLs across the next $\hat{a}$ years. The consequences of the two World Wars in the 20^{th} century generate essentially similar patterns, with large spikes in the lifespan component of GD and a much more sluggish reaction in ID.

Note also that, had France returned to a stationary population after 1870, we would expect ID and GD to converge in 1910, 40 years after the end of the war. ID, however, remains well above GD, a direct consequence of the dramatic fall in mortality rates, particularly among infants, that started at the end of the 19^{th} century (see the right panel of Figure 5). This fall corresponds to a succession of (negative) permanent shocks in mortality rates, which GD mirrors instantaneously while ID takes decades to reflect.

Finally, while we would expect ID in 1919 to remain at the levels of the 1914-1918 period due to its inertia, we observe a discrete fall. This is due to the changing status of the Alsace-Lorraine region. After 1918, Alsace-Lorraine, which was German since 1870, became French again. This implies that all the deaths that occurred in that region between 1870 and 1918 were not recorded by the French administration and are therefore not reported in the PYPLs we measure. (These deaths were simply not observed and were attributed to Germany.) This explains the discrete drop in ID in 1919: a large living population is added to France in 1919 but the death history of this population is not.[15] This example illustrates the difficulty in implementing ID in practice. To be consistent, ID requires stable geographical units over long enough periods, while countries borders often change (think of the USSR in the 1990s).

3 Characterization of the two indices

In this section, we provide a formal characterization of ID and GD. Excluding trivial distributions for which no individual is alive or prematurely dead, the set of distributions in period t is denoted as:

$$\mathcal{X} = \{\mathbf{x} \in \cup_{n\in\mathcal{N}}(\mathcal{Z} \times \mathcal{S})^n \mid \text{there is } i \text{ for whom either } s_i \neq D \text{ or } s_i = D \text{ and } \hat{a} > t - b_i\}.$$

3.1 Inherited deprivation

We first show that ID is characterized by a small number of desirable properties. First, indices based exclusively on alive deprivation satisfy a property which requires that the presence of an additional dead individual, whether prematurely or not, does not affect them. We refer to this property as:

Deprivation axiom 1 (Independence of Dead). *For all $\mathbf{x} \in \mathcal{X}$ and $i \leq n(\mathbf{x})$, if $s_i = D$, then $P(\mathbf{x}_i, \mathbf{x}_{-i}) = P(\mathbf{x}_{-i})$.*

By contrast, we require that the presence of an additional dead individual does not affect ID only if this individual is born at least $\hat{a}$ years before period t:

Deprivation axiom 2 (Weak Independence of Dead). *For all $\mathbf{x} \in \mathcal{X}$ and $i \leq n(\mathbf{x})$, if $s_i = D$ and $\hat{a} \leq t - b_i$, then $P(\mathbf{x}_i, \mathbf{x}_{-i}) = P(\mathbf{x}_{-i})$.*

The second property, Least Deprivation, requires that being non-poor is better than being either poor or prematurely dead. This weak axiom compares distributions with a unique individual, i.e. individual 1, in which, if the individual is dead, she is prematurely dead.

Deprivation axiom 3 (Least Deprivation).
$P(b_1, NP) < P(b_1, AP)$ and $P(b_1, NP) < P(b_1, D)$.

The third property, Weak Independence of Birth Year, requires that birth years are only relevant in order to distinguish prematurely dead from other dead individuals.[16] Hence, if an individual belongs to the reference population, only her status matters.

[15] Note that the annexation of Alsace Lorraine by Germany in 1870 also affected ID during the 1870-1910 period: all deaths occurring prior to 1870 in that region are still attributed to France afterwards, while the living population is not. During the 1870-1910 period, ID is systematically overestimated. The same is true when Alsace-Lorraine is annexed again by Germany during World War II.

[16] This is why Weak Independence of Birth Year has the precondition $d(\mathbf{x}_i, \mathbf{x}_{-i}) = d(\mathbf{x}'_i, \mathbf{x}_{-i})$, which holds the number of prematurely dead constant: the birth year b'_i can be different from b_i, but if $s_i = D$, then individual i is either prematurely dead in both $\mathbf{x}_i$ and $\mathbf{x}'_i$, or in none of these two bundles.

Deprivation axiom 4 (Weak Independence of Birth Year). *For all* $\mathbf{x} \in \mathcal{X}$ *and* $i \leq n(\mathbf{x})$, *if* $s_i = s'_i$ *and* $d(\mathbf{x}_i, \mathbf{x}_{-i}) = d(\mathbf{x}'_i, \mathbf{x}_{-i})$, *then* $P(\mathbf{x}_i, \mathbf{x}_{-i}) = P(\mathbf{x}'_i, \mathbf{x}_{-i})$.

Weak Independence of Birth Year requires that one person-year prematurely lost matters equally in the index, independently of the particular age of the individual who died. Thus, if $\hat{a}$ is equal to 50, the death of a newborn in $t-1$ is equivalent to the death of a 48 years old in $t-1$ in the computation of ID at period t. Of course, the death of the younger individual will be recorded in the ID indices for several periods following her death, while the death of the 48 years old individual will be accounted for only once (in the period t following her death). In that sense, the death of the younger individual matters proportionally more.

Then, we impose a standard separability property, Subgroup Consistency. This axiom requires that, if deprivation decreases in a subgroup while remaining unchanged in the rest of the distribution, overall deprivation must decline.[17]

Deprivation axiom 5 (Subgroup Consistency). *For all* $(\mathbf{x}, \mathbf{x}'), (\mathbf{x}, \mathbf{x}'') \in \mathcal{X}$, *if* $P(\mathbf{x}') > P(\mathbf{x}'')$ *and* $f(\mathbf{x}')+p(\mathbf{x}')+d(\mathbf{x}') = f(\mathbf{x}'')+p(\mathbf{x}'')+d(\mathbf{x}'')$, *then* $P((\mathbf{x}, \mathbf{x}')) > P((\mathbf{x}, \mathbf{x}''))$.

To be complete, three auxiliary properties are also needed: Anonymity, Replication Invariance and Young Continuity. First, the name of individuals should not influence the deprivation index.

Deprivation axiom 6 (Anonymity). *For all* $\mathbf{x} \in \mathcal{X}$, *if* $n(\mathbf{x}') = n(\mathbf{x})$ *and* $\mathbf{x}'$ *is obtained from* $\mathbf{x}$ *by a permutation of the index set* $\{1, \dots, n(\mathbf{x})\}$, *then* $P(\mathbf{x}) = P(\mathbf{x}')$.

Second, if a distribution is obtained by replicating another distribution several times, they both have the same deprivation. For any $k \in \mathcal{N}$, we denote by $\mathbf{x}^k$ the k-**replication** of $\mathbf{x}$, which is the distribution such that $n(\mathbf{x}^k) = kn(\mathbf{x})$ and $\mathbf{x}^k = (\mathbf{x}, \mathbf{x}, \dots, \mathbf{x})$.

Deprivation axiom 7 (Replication Invariance). *For all* $\mathbf{x} \in \mathcal{X}$ *and* $k \in \mathcal{N}$, $P(\mathbf{x}^k) = P(\mathbf{x})$.

Finally, the deprivation index evolves "continuously" on its domain. Given that this domain is discrete, the index should satisfy a particular continuity property as proposed by Young (1975).

Deprivation axiom 8 (Young Continuity). *For all* $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathcal{X}$, *if* $P(\mathbf{x}) > P(\mathbf{y})$ *and* $n(\mathbf{z}) = 1$, *then for* $k \in \mathcal{N}$ *sufficiently large we have* $P(\mathbf{x}^k, \mathbf{z}) > P(\mathbf{y})$ *and* $P(\mathbf{x}) > P(\mathbf{y}^k, \mathbf{z})$.

Proposition 1 fully characterizes ID, which implies that any deprivation index satisfying our properties ranks distributions in exactly the same way as ID.

Proposition 1 (Characterization of ID).
P is ordinally equivalent to ID_γ for some $\gamma > 0$ if and only if P satisfies Weak Independence of Dead, Least Deprivation, Weak Independence of Birth Year, Subgroup Consistency, Anonymity, Replication Invariance and Young Continuity.

[17] The precondition $f(\mathbf{x}') + p(\mathbf{x}') + d(\mathbf{x}') = f(\mathbf{x}'') + p(\mathbf{x}'') + d(\mathbf{x}'')$ ensures that distributions $\mathbf{x}'$ and $\mathbf{x}''$ have a reference population with the same size. The additive separability result of Foster and Shorrocks (1991), which rationalizes the use of additive indices, is based on a stronger version of Subgroup Consistency with the additional precondition $f(\mathbf{x}') + p(\mathbf{x}') = f(\mathbf{x}'') + p(\mathbf{x}'')$.

Proof. See Online Appendix. $\square$

Proposition 1 provides a necessary step in the characterization of GD. Observe that our definition of the individual status is agnostic to the particular definition of alive deprivation, and could as well capture income deprivation, as in our empirical application, or multidimensional poverty (Alkire and Foster, 2011). Proposition 1 can be extended to a framework in which alive deprivation is measured as a continuous variable such as an income deprivation score or a multidimensional poverty score, provided that the axioms are duly adapted (see Foster and Shorrocks (1991)).

3.2 Generated deprivation

We focus here on current mortality, and define the set of mortality vectors as:

$$\mathcal{M} = \left\{ \mu \in [0,1]^{a^*+1} \middle| \mu_{a^*} = 1 \right\}.$$

We consider pairs $(\mathbf{x}, \mu)$ for which the distribution $\mathbf{x}$ is a priori unrelated to vector μ. We assume that the age-specific mortality rates μ_a must be feasible given the number of alive individuals $n_a(\mathbf{x})$. Given that distributions have finite numbers of individuals, mortality rates cannot take irrational values, i.e. $\mu_a \in [0,1] \cap \mathcal{Q}$, where $\mathcal{Q}$ is the set of rational numbers. The set of pairs considered is[18]

$$\mathcal{O} = \left\{ (\mathbf{x}, \mu) \in \mathcal{X} \times \mathcal{M} \middle| \text{for all } a \in \{0, \dots, a^*\} \text{ with } n_a(\mathbf{x}) > 0 \text{ we have } \mu_a = \frac{c_a}{n_a(\mathbf{x})} \text{ for some } c_a \in \mathcal{N} \right\}.$$

An index is a function $\mathbf{P} : \mathcal{O} \to \mathcal{R}_+$.

Our characterization above argues that, when measuring deprivation using past mortality, ID is the appropriate measure. As current mortality is always the same as past mortality in stationary populations, we therefore require that any index defined on current mortality rates is equivalent to ID in the case of stationary populations.[19] Let $\mathcal{O}^*$ denote the subset of all pairs in $\mathcal{O}$ that are stationary.

Deprivation axiom 9 (ID Equivalence). *There exists some $\gamma > 0$ such that for all $(\mathbf{x}, \mu) \in \mathcal{O}^*$ we have $\mathbf{P}(\mathbf{x}, \mu) = ID_\gamma(\mathbf{x})$.*

Besides ID Equivalence, GD is characterized by two desirable properties. First, GD satisfies Additive Decomposibility, a strengthening of Subgroup Consistency. This property implies that, if deprivation decreases in a subgroup while remaining unchanged in the rest of the population, overall deprivation declines.

Deprivation axiom 10 (Additive Decomposibility). *For all $(\mathbf{x}', \mu'), (\mathbf{x}'', \mu'') \in \mathcal{O}$, if $\mathbf{x} = (\mathbf{x}', \mathbf{x}'')$ and $\mu_a = (n_a(\mathbf{x}') * \mu'_a + n_a(\mathbf{x}'') * \mu''_a)/(n_a(\mathbf{x}') + n_a(\mathbf{x}''))$ for all $a \in \{0, \dots, a^*\}$, then*

$$\mathbf{P}(\mathbf{x}, \mu) = \frac{\eta(\mathbf{x}', \mu') * \mathbf{P}(\mathbf{x}', \mu') + \eta(\mathbf{x}'', \mu'') * \mathbf{P}(\mathbf{x}'', \mu'')}{\eta(\mathbf{x}', \mu') + \eta(\mathbf{x}'', \mu'')}, \tag{4}$$

where the "size" function $\eta : \mathcal{O} \to \mathcal{N}_0$ is such that $\eta(\mathbf{x}, \mu) = \eta(\mathbf{x}', \mu') + \eta(\mathbf{x}'', \mu'')$.

[18] To be complete, the definition of $\mathcal{O}$ is such that for all $a \in \{0, \dots, a^*\}$ with $n_a(\mathbf{x}) = 0$ we have $\mu_a = 0$ when $a < a^*$ and $\mu_{a^*} = 1$.

[19] Recall that past mortality is recorded in distribution $\mathbf{x}$ while current mortality is recorded in vector μ. As vector μ is redundant in stationary pairs, the index can be computed on distribution $\mathbf{x}$ only. Proposition 1 then argues that the index should be equal to ID_γ.

Additive Decomposibility implies that the index is decomposable in subgroups. A decomposable index measured on a set of individuals can always be calculated as the weighted sum of the same index measured on any partition of this set, where the weight attributed to a subset is the fraction of its reference population divided by the total reference population. This property matters if one wishes to compare the relative deprivation of different groups in a society, such as men and women or black and white individuals.

Second, GD does not directly depend on mortality prior to period t and satisfies instead Independence of Dead* (where the asterisk denotes that Independence of Dead is adapted to domain $\mathcal{O}$).

Deprivation axiom 11 (Independence of Dead*). *For all* $(\mathbf{x}, \mu) \in \mathcal{O}$ *and* $i \leq n(\mathbf{x})$, *if* $s_i = D$, *then* $\mathbf{P}((\mathbf{x}_i, \mathbf{x}_{-i}), \mu) = \mathbf{P}(\mathbf{x}_{-i}, \mu)$.

Proposition 2 shows that these three properties jointly characterize GD.

Proposition 2 (Characterization of GD).
$\mathbf{P} = GD_\gamma$ *for some* $\gamma > 0$ *if and only if* $\mathbf{P}$ *satisfies ID Equivalence, Independence of Dead* and Additive Decomposibility.*

Proof. See Online Appendix. □

We conclude this section by discussing a particular feature of GD. By definition, the reference population of GD depends on premature mortality. The larger the premature mortality associated with vector μ, the larger GD's reference population. Such is not the case for ID, since (past) premature mortality changes the *status* of individuals in ID's reference population, but not its size. Given that its reference population depends on premature mortality, GD may violate the following natural requirement: for a fixed alive deprivation, if lifespan deprivation increases, total deprivation should not decrease. We therefore require GD to satisfy Monotonicity in Current Mortality.

Deprivation axiom 12 (Monotonicity in Current Mortality). *For all* $(\mathbf{x}, \mu), (\mathbf{x}, \mu') \in \mathcal{O}$, *if* $\mu_a \geq \mu'_a$ *for all* $a \in \{0, \ldots, \hat{a} - 2\}$, *then* $\mathbf{P}(\mathbf{x}, \mu) \geq \mathbf{P}(\mathbf{x}, \mu')$.

An implication of Monotonicity in Current Mortality is that GD cannot attribute a lower weight to one PYPL than to one PYAD:

Proposition 3. GD_γ *satisfies Monotonicity in Current Mortality if and only if* $\gamma \geq 1$.

Proof. See Online Appendix. □

Note that, as shown in the proof for Proposition 3, the condition under which GD is monotonic in mortality rates for a given society requires the weight γ to be at least as large as the society's head-count ratio (HC).[20] As a result, when alive deprivation is moderate, the constraint on γ is less restrictive.

[20] This is not in contradiction with the stronger necessary condition stated in Proposition 3. This proposition provides the condition under which GD is monotonic *for all* pairs $(\mathbf{x}, \mu)$ in $\mathcal{O}$, some of which have a HC equal to one.

4 Comparison with alternative approaches

Several measures have been proposed in the literature to combine basic welfare with mortality indicators into a single index. In this section we compare our deprivation indices to these alternative measures. This allows us to discuss some important assumptions underlying the construction of our indices.

4.1 Composite indices

The first approach is to use *composite* indices such as the Human Development Index. This simple indicator of well-being aggregates mortality with income information as a weighted sum of its mortality and income components, typically using equal weights. As discussed in Ravallion (2012*b*), this type of aggregation hides underlying trade-offs between the dimensions being aggregated. More fundamentally, as shown in the Introduction, a composite deprivation index $\mathbf{P}_w$ is inconsistent with a basic separability property, as it does not assign a fixed relative weight to one PYPL compared to one PYAD. In particular, when allowing some individuals to live longer in alive deprivation instead of dying prematurely, the index $\mathbf{P}_w$ may increase or decrease, depending on the fraction of the living population which is initially income poor.[21]

Closely related to the indices proposed in this paper, the Human Poverty Index (HPI) is a composite index that aggregates both premature mortality and alive deprivation (Watkins, 2006). Its premature mortality component $HC_{\hat{a}}$ measures the probability that a newborn dies before turning $\hat{a}$ years. The HPI is defined as a weighted average of alive deprivation, HC, and lifespan deprivation thus defined, $HC_{\hat{a}}$:

$$HPI_w(\mathbf{x}, \mu) = w * HC(\mathbf{x}) + (1 - w) * HC_{\hat{a}}(\mu),$$

with $w \in [0, 1]$. One can easily adapt the example given in the Introduction to show that the HPI suffers from the same inconsistency as index $\mathbf{P}_w$, even though HPI does not measure premature mortality in time-units.[22]

The inconsistency affects all composite indices as soon as their components have different reference populations. In the case of $\mathbf{P}_w$, its alive deprivation component divides the number of PYADs by the number of PYs spent *alive* while its mortality component divides the number of PYs spent alive by the normative lifespan. The implicit weight that this index attaches to one PYAD over one PYPL therefore depends on the relative levels of alive deprivation and life expectancy. The root of the problem is that $\mathbf{P}_w$ first normalizes each component using different reference populations, before summing them. In contrast, our total deprivation indices add the number of PYADs to the number of PYPLs *before* normalizing by the same reference population. As a result, the relative weight attributed to one PYPL over one PYAD

[21] This problem does not depend on the value of the parameter w. For all possible values of the $w \in (0, 1)$, one can always find situations under which the composite index $\mathbf{P}_w$ is not consistent.

[22] Consider for instance three stationary societies: A, B and C. Four individuals are born every year. In society A, all individuals live until they reach old age. Two individuals live their whole life in poverty and the other two are never poor. $HC(A) = 0.5$, $HC_{\hat{a}}(A) = 0$ and $HPI_{0.5}(A) = 0.25$. Society B is the same as society A, except that one poor individual dies in early childhood. $HC(B) = 0.33$, $HC_{\hat{a}}(B) = 0.25$ and $HPI_{0.5}(A) = 0.29$. Society C is the same as society A, except that the two poor individuals die in early childhood. $HC(C) = 0$, $HC_{\hat{a}}(C) = 0.5$ and $HPI_{0.5}(C) = 0.25$.

remains constant. Moreover, the value of this weighing parameter can be chosen normatively, in a meaningful and transparent way. Another difference between our indices and composite indices is that our total deprivation indices generalize the alive deprivation index. In the absence of premature mortality, they are identical to alive deprivation, as measured by HC.

4.2 Preference-based indicators and the choice of γ

A second approach is to use preference-based indicators that aggregate the quality and quantity of life by assuming or calibrating a particular inter-temporal utility function, unique across time and space (Becker, Philipson and Soares, 2005; Grimm and Harttgen, 2008; Jones and Klenow, 2016). These indicators are partly based on the actual achievements of non-poor individuals, which our deprivation indices disregard. Moreover, these approaches implicitly attribute values to one year of extra life that vary with the living standards of the country, reflecting the higher opportunity cost of dying in richer countries. This property is shared by some composite measures of well-being, such as the Human Development Index (HDI). As shown by Ravallion (2012*b*,*a*), the implicit value of one extra year of life given by the HDI is typically larger in richer countries.

Our indicators instead aggregate alive and lifespan deprivation without relying on a particular representation of the preferences. From the perspective of the practitioner, they require fewer normative assumptions and essentially rely on selecting values for two transparent normative parameters: the age threshold $\hat{a}$ and γ, the fixed weight parameter. Also, in our deprivation framework, there is at the individual level no reason to trade-off differently one PYAD over one PYPL depending on the observed levels of alive deprivation and life expectancy: a fixed weight γ is a natural requirement. Fundamentally, the two dimensions we compare in computing these indices are deprivation statuses (i.e. being poor or being prematurely dead) instead of "achievements" (such as mean income or life expectancy). Under a common utility function, a given deprivation status (AP or D) leads to a fixed level of instantaneous utility, regardless of the country.[23] As a result, at the individual level, a given deprivation status carries the same weight in all countries, and the trade-off between two deprivation statuses is the same across countries. Of course, a country could become increasingly averse to lifespan deprivation (or poverty) as its level increases, implying that its normative weight would depend on this level.[24] Here, such judgments are ruled-out by Subgroup Consistency, which forces the deprivation index to use the same normative weight on the whole population as that used on subgroups. As an individual can constitute a subgroup, this property forces the index to use a constant weight, regardless of the level of lifespan deprivation (or poverty). In other words, having a constant weight is necessary if one wishes to compare the relative deprivation of different groups in a society (Additive Decomposibility).

As the above discussion illustrates, parameter γ is conceptually different from the HDI's trade-off between mean income and life expectancy. However, they are not entirely disconnected. Even if neither mean income nor life expectancy have intrinsic

[23] For instance, two individuals living in different countries are assumed to have the same (low) utility if they are both under the extreme poverty line of the World Bank.

[24] As pointed out by a referee, in the case of GD, such society would have its parameter γ depend on $p(\mathbf{x})$ or $d^{GD}(\mathbf{x}, \mu)$.

value in our indices, they indirectly affect alive deprivation and lifespan deprivation. For instance, poverty typically decreases non linearly with increases in mean income, whereas lifespan deprivation typically decreases non linearly with increases in life expectancy. Using an estimate of these non-linear functions, one could in principle compute the average loss in mean income that leaves our index unaffected when life-expectancy is increased by one year. Such a monetary estimate of the value of one extra year of life depends directly on parameter γ. If γ is close to zero, then the monetary estimate is close to zero, and when γ tends to infinity, so does this estimate.

4.3 Deprivation measures improving on the mortality paradox

The third approach keeps an exclusive focus on poverty but "corrects" poverty measures for the higher mortality rates affecting low income groups. Kanbur and Mukherjee (2007) argue that such selective mortality leads to serious mis-measurement of income poverty. Indeed, higher mortality rates among the poor lead to a "mortality paradox", whereby poor who died early are ignored. A central objective of that literature is to design poverty measures that explicitly takes this into account. The idea is to remove the *instrumental* impact of selective mortality on standard poverty measures by assigning fictitious incomes to the prematurely dead individuals Kanbur and Mukherjee (2007), Lefebvre, Pestieau and Ponthiere (2013, 2017). The validity of these approaches relies on the assumptions made in the construction of these counterfactual, "fictitious" incomes.

In this perspective, premature mortality is not *intrinsically* valued, but only taken into account when associated with poverty. Our approach is fundamentally different, as we consider premature death as an intrinsic form of deprivation. We do not think that the income an individual would have earned had she remained alive is relevant to quantify the total deprivation experienced by a society. Consider two populations, A and B, which are identical except that, in A, the prematurely dead individuals would have been poor had they lived, but not in B. By construction, alive deprivation and lifespan deprivation are the same in both societies. In our view, the total deprivation experienced in these two societies is identical, while the measures addressing the mortality paradox would systematically consider population A as poorer.

To take another example, consider a poor population made of two subgroups of equal size, say men and women. Both women and men are poor in the first period. If they survive to the second period, women become non-poor but men stay poor. In scenario A, all women die at the end of the first period, while all men survive. In scenario B, all men die at the end of the first period, while all women survive. Premature mortality is the same in the two scenarios, but there is more poverty in A because men survive. While we would conclude that deprivation is higher in scenario A, poverty measures correcting for the mortality paradox would typically assess the same levels of poverty across the two scenarios. In other words, these measures allow the higher counterfactual poverty of the prematurely dead to outweigh the lower poverty levels of the living.

Clearly, poverty is certainly a cause of premature mortality (and vice-versa), and policy recommendations should certainly take these causal relationships into account.

However, we do not believe that these *positive* relationships matter for the *normative* comparison of deprivation outcomes. Here is an analogy. An individual derives the utility $U(b, f)$ from the number of bees (b) and the number of flowers (f) in his garden. In practice, the number of flowers affects the number of bees, and vice versa. Yet, the normative evaluation of the garden, i.e. the function U, does not take into account the causal links between b and f.

The literature on the mortality paradox proposes various methods to assign fictitious incomes to missing individuals. One such method assigns fictitious incomes regardless of the pre-mortem income of missing individuals (e. g. Lefebvre, Pestieau and Ponthiere (2013, 2017)). This idea can in principle be applied in our constrained information setup. However the definition of a missing poor used there is conceptually very different from ours as it is based on a reference mortality vector, corresponding to that of the most affluent societies such as Norway or the US: the missing population is defined as those individuals who died *in excess* with respect to this reference mortality vector. As a result, not all individuals dying prematurely are considered as missing individuals while an 80-year-old individual dying in excess would. Our deprivation approach focusses on all premature deaths and, therefore, does not rely on a notion of excess mortality relative to a reference vector.

Alternatively, one could assign fictitious incomes that depend on the incomes earned before dying. Thus, Kanbur and Mukherjee (2007) attribute to rich individuals dying prematurely fictitious incomes that are above the deprivation threshold. In our approach, we do not distinguish between the premature mortality affecting the poor and that affecting the non-poor. As noted in the Introduction, the necessary information on the mortality rates of different income groups is often not available.

More fundamentally, a normative issue raised by the literature on multidimensional poverty (Bourguignon and Chakravarty, 2003) is that there is more overall poverty if the same individuals concentrate several dimensions of deprivation. The premature mortality of poor individuals constitutes such a non-desirable concentration of deprivations. To address this question, we need to distinguish mortality rates between poor and non poor individuals. To make our indices sensitive to concentration, we can for instance define an individual as being in *total poverty* if she spends more than k person-years in deprivation, either in the form of PYPLs or PYADs. Our indices can therefore be accommodated to allow for this type of approach.[25] However, to compute such concentration-sensitive indices of total poverty, we need not only mortality rates by income groups but also information on *mobility in and out* alive deprivation across consecutive periods, a type of information which is typically not available.[26]

5 Measuring deprivation

In this section, we apply our indices of total deprivation for low- and middle-income countries for the period of 1990 to 2015. Our objective is to characterize the level of

[25] Such a definition of total poverty is consistent with the definition of multidimensional poverty proposed by Akire and Foster (2011): an individual is multidimensionally poor if she is deprived in at least k dimensions.

[26] Note also that, when mobility is very low and premature mortality is mostly concentrated on poor individuals, our indices approximately count the number of person-years lost to deprivation by the poor.

deprivation worldwide, how it differs across countries, and how it has changed over time, as well as to understand how these patterns, based on total deprivation, differ from those based on a more standard poverty measure such as the headcount ratio (HC).

ID requires information on the number of death by age in the past $\hat{a}$ years. Such information exists, for example in the Human Mortality Database, but the countries available in this database are very different from those for which comparable alive deprivation data is available or for which deprivation measures would be relevant. In addition, as illustrated in Subsection 2.4.3, ID also requires country borders to remain stable in the last $\hat{a}$ years to be meaningful, while the 20^{th} century has seen considerable changes in countries borders and in the number of countries. For these reasons, ID is ill-suited for our empirical exercise, and we focus in the following on GD, which we consider as the most relevant index in practice, given the data constraints.

5.1 Data

The definition of our indices requires a value for the age threshold $\hat{a}$ and a weight γ. As already discussed, the latter will be set conservatively at 1, so that one person-year prematurely lost is equivalent to one person-year spent in income deprivation. Choosing a higher value for γ, by increasing the weight given to the mortality component, would simply magnify the difference with respect to HC.

The choice of the age threshold is analogous to the choice of an income threshold used for income deprivation. It is ultimately a normative choice about the minimum number of years of life that a society judges essential for its members. In the following, we use a threshold $\hat{a} = 50$ years, which is much lower than the median age at death observed in our data (64 years old). Of course, a higher age threshold would inflate our indices and their difference with income deprivation measures.[27]

The computation of GD requires information on alive deprivation as well as information on mortality by age in the period under study. In the following, we make use of two publicly available data sets to construct our measures of deprivation. The data on population and mortality by country, age group and year comes from the Global Burden of Disease database (2017 version of the data) (Global Burden of Disease Collaborative Network, 2018). Comparable information across countries and over time is available for the 1990-2017 period and is, to our knowledge, the most comprehensive mortality data available for international comparison. To construct this database, population and mortality data are systematically recorded across countries and time from various data sources (official vital statistics data, fertility history data as well as data sources compiling deaths from catastrophic events). These primary data are then converted into data at the age group, year and country level using various interpolations and inference methods (see Dicker et al. (2018) and Murray et al. (2018) for more information on the GBD data construction).[28]

[27]In the code available online, the reader can compute deprivation indices choosing various values of $\hat{a}$ and γ, as well as alternative income poverty thresholds.

[28]The number of deaths in each cell is an estimate and comes with a confidence interval. Following the convention in the literature, we do not use these confidence intervals, and only consider the point estimate of the number of death (see also Hoyland, Moene and Willumsen (2012) for a critique of this approach). Moreover, the mortality information is given into 5 year age brackets (except for the 0-5 years group, for which the information is decomposed into 0-1 and 1-5). When necessary,

Data on alive deprivation come from the PovcalNet website (World Bank's Development Research Group, 2019) which provides internationally comparable estimates of income deprivation level. This data set is based on income and consumption data from more than 850 representative surveys carried out in 127 low- and middle-income countries between 1981 and 2015.[29] Each country's income deprivation level in PovCalNet is computed on a three year basis, so that the yearly data used below were obtained by a linear interpolation of income deprivation estimates across years. A complete description of the data set is given in Chen and Ravallion (2013).[30] In our empirical application, we follow the World Bank's definition of extreme income deprivation, corresponding to the 1.90$ a day threshold (Ferreira et al., 2016). We merged the two databases at the year and country level. Since the Global Burden of the Disease data are only available since 1990 and the PovCalNet data until 2015, we focus on the 1990-2015 period for a total of 113 low- and middle-income countries (see Online Appendix 2 for a list of those countries).

5.2 Worldwide deprivation

GD is defined on the total number of years of deprivation generated in a given year. Table 3 presents this computation for the World in 1990 and 2015. In 2015, 1,105 million person-years of deprivation have been generated, 703 million from income deprivation and 402 from lifespan deprivation. Relative to the reference population (that is, 6,010 million alive individuals to which we add 402 million person-years lost to lifespan deprivation), this implies that 17.2% of the person-years in 2015 were lost to deprivation. This is much lower than the deprivation level of 52.8% measured in 1990.

Figure 6 presents the evolution of world's total deprivation (GD), and its two components, alive and lifespan deprivation. We also report the HC ratio for comparison purposes.[31] First, GD and HC follow a similar trend and do not offer a different diagnostic about the evolution of world deprivation in the last 25 years. World deprivation fell dramatically between 1990 and 2015.

However, lifespan deprivation is far from negligible. In 1990, it represented 27% of total deprivation. This number is a direct measure of the underestimation of deprivation when premature mortality is not taken into account. Even though lifespan deprivation declined over this period, its relative importance increased over time: its share in total deprivation increased from 27% in 1990 to more than 36% in 2015. Given our conservative choice of parameters, these estimates can be considered as a lower bound. For example, if $\hat{a}$ is set at 80 years, total deprivation in 2015 would be underestimated by more than 60%. The increase over time in the share of lifespan deprivation indicates that more progress has been made against alive deprivation as against lifespan deprivation over the past 25 years. One can only wonder if this

we transform the data into age groups of one year by assuming a uniform death rate within an age category. Finally, the older age group is "95 and above". As we do not know the precise age of death of individuals in that category, we assume that 95 is the maximum age they can reach. This last assumption is of no consequence, since our age threshold $\hat{a}$ is well below 95.

[29] The website address is http://iresearch.worldbank.org/PovcalNet/povOnDemand.aspx.

[30] Clearly, these transformations may matter for the empirical analysis, as they tend to smooth the evolution of income deprivation across years. In particular, in the case of catastrophic events, income deprivation appears as less reactive then lifespan deprivation, which may be due to the interpolated nature of the income deprivation data.

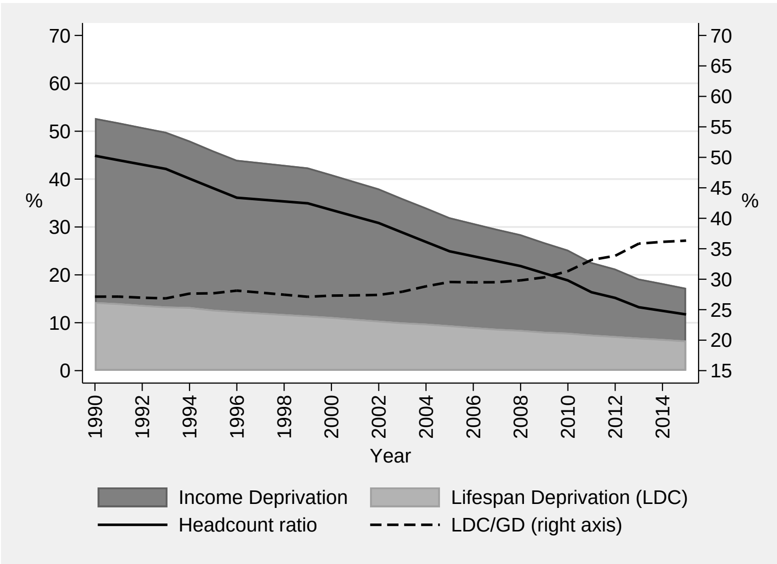
[31] Strictly speaking, HC and GD can not be directly compared, since they are based on different reference populations.

Table 3: Generated Deprivation in the developing world in 1990 and 2015, with $\hat{a} = 50$.

	Unit	1990 Value	2015 Value	Computation
Living population	person-years (million)	4,200	6,010	Source: GBD (2017)
HC	%	44.9	11.7	Source: PovCalNet
Alive Deprivation (PYAD)	person-years (million)	1,886	703	Living population * HC
Lifespan Deprivation (PYPL)	person-years (million)	701	402	See Equation (3)
Deprived population	person-years (million)	2,587	1,105	PYAD+PYPL
Reference population	person-years (million)	4,901	6,412	Living population + PYPL
GD_1	%	52.8	17.2	$\frac{Deprived\ Population}{Reference\ Population}$

would have been the case had premature mortality systematically been taken into account in deprivation measures.

Figure 6: Decomposition of Generated Deprivation with $\hat{a} = 50$, World Level

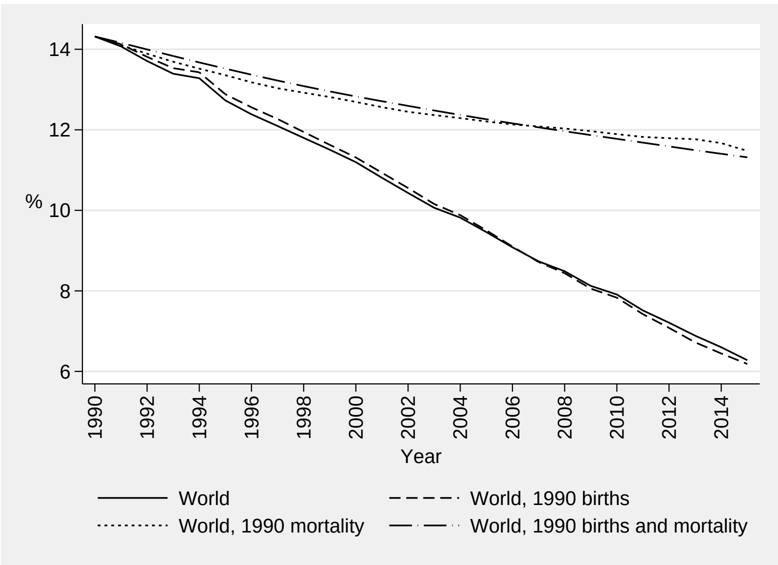


Child mortality is a major component of lifespan deprivation: the deaths of children aged 5 or below account for almost 80% of GD's lifespan component in 1990 and close to 70% in 2015. This is due to the fact that (i) mortality rates of this population are high relative to other age cohorts and (ii) the number of years prematurely lost generated by deaths at younger ages is large (a newborn death generates exactly $\hat{a}$ person-years lost).

Three potential mechanisms could be at work behind the observed decline in lifespan deprivation between 1990 and 2015. First, natality rates could have fallen: given that young children have a relatively larger mortality rate, this leads to a decrease in deprivation, for given levels of mortality rates. Alternatively, mortality rates could have decreased, holding natality constant. Finally, the changes in lifespan deprivation could be due to changes in the relative size of each cohort resulting from the changes in natality and mortality rates that occurred before 1990.

Figure 7 illustrates the relative importance of these mechanisms using three different counterfactuals. In the first scenario, we maintain constant the number of births to its 1990 level, which neutralizes the effects of a change in natality rate. In the second scenario, we keep the age-specific mortality rates at their 1990 level, and in the third, we keep both the 1990 natality and age-specific mortality rates to concentrate on the changes in the population pyramid. Comparing the World curve (in solid line) to its various counterfactuals, the main drivers of the evolution in lifespan deprivation are both the fall in mortality rates and the changes in the composition of the population pyramid. Mortality rates, particularly among newborns, fell dramatically from 6.6% in 1990 to 3.4% in 2015.[32] Changes in the population pyramid is the other important contributor, indicating that changes in mortality and natality prior to 1990 led to a long run decline in the share of the cohorts with larger mortality rates. By contrast, natality rates remained essentially stable over the period (they increased by 2.8% between 1990 and 2015), and do not contribute to the observed changes in lifespan deprivation.

Figure 7: Counterfactual Worlds: evolution of GD's lifespan deprivation component, with $\hat{a} = 50$



5.3 Country level deprivation

5.3.1 The level of deprivation

We now examine deprivation at the level of individual countries. The parallel evolution of GD and HC at the world level may indeed hide large differences at less aggregated levels. Figure 8 provides an example which compares alive deprivation

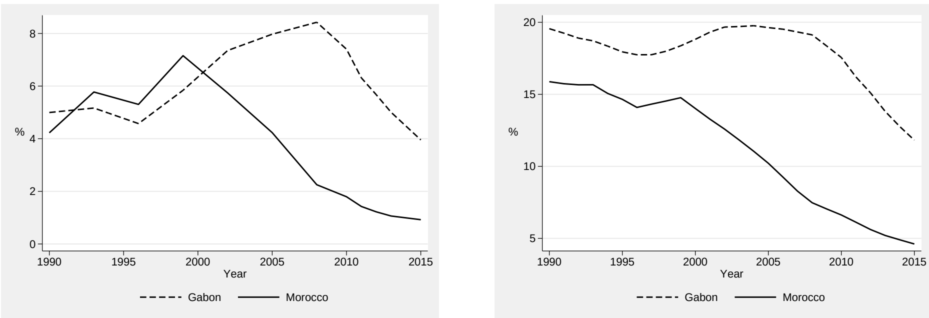
[32] Note that since this amounts to a succession of permanent negative mortality shocks, we expect ID to be larger than GD between 1990 and 2015, as discussed in Section 2.4.2.

(HC) to total deprivation (GD) in Morocco and Gabon. According to HC, throughout the 1990s, Gabon and Morocco are virtually at the same level of alive deprivation. However, total deprivation is much higher in Gabon once lifespan deprivation is taken into account.

Individual countries are often compared and ranked according to their level of deprivation (Hoyland, Moene and Willumsen, 2012). On the basis of income deprivation (starting from the least poor), Gabon is 19^{th} and Morocco 21^{th} in 1993. If one uses total deprivation (GD) instead, Gabon is 38^{th} and Morocco 32^{th}. That's a difference of 9 ranks out of 113 countries. Table 4 decomposes the sources of this re-ranking in 1993. While both countries have a similar level of alive deprivation (with a value of HC around 5.5%), mortality rates are much higher in Gabon. Thus, in 1993, life expectancy at birth in Morocco is 67 years, against 59 years in Gabon.

More generally, some countries are actually much more deprived than originally thought on the basis of HC, and the use of GD leads to substantial re-rankings across countries. The average difference in ranking when applying GD instead of HC across all the 113 countries analyzed is equal to 4.1 ranks throughout the period. In particular, countries of the ex-USSR and a few African countries are much more deprived than indicated by the HC ratio, while the ranks of Latin American countries improve substantially. As the relative importance of the lifespan component increases, these re-rankings have become larger over time, from 4.4 in 1990 to 4.9 in 2015. The largest re-ranking is that of Azerbaijan, which loses 26 ranks in 2010 and 2014: while alive deprivation almost disappeared in Azerbaijan, its population still faces relatively large levels of lifespan deprivation.

Figure 8: Examples of re-rankings: Gabon and Morocco. HC and GD with $\hat{a} = 50$



Head-count ratio

Total Deprivation

5.3.2 The evolution of deprivation

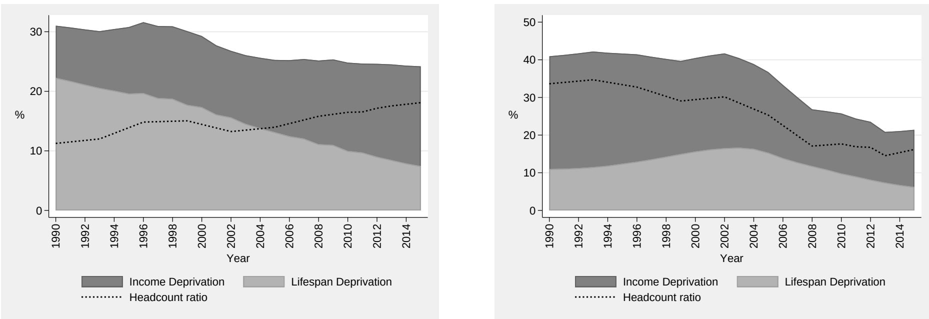
GD can also be used to assess the evolution of deprivation in a given country. Consider the cases of the Comoros and Botswana. Figure 9 illustrates the contrasting evolutions of HC and GD in the case of the Comoros for the period 1990-2015 and Botswana during the 1990s.

According to HC, alive deprivation increased in the Comoros by 60% between 1990 and 2015. However, over the same period, GD fell by 23%. Focusing on alive deprivation hides the large progress made in lifespan deprivation: while the number of PYADs increased more rapidly than the population, the number of PYPLs decreased drastically during the period. Overall, the total number of person-years lost to both

Table 4: An example of re-ranking. Decomposition of GD in Gabon and Morocco in 1993.

	Unit	Gabon	Morocco	Computation
Living population	person-years (thousand)	1,043	26,449	Source: GBD (2017)
HC	%	5.2	5.8	Source: PovCalNet
Alive Deprivation (PYAD)	person-years (thousand)	54	1,534	Living population * HC
Lifespan Deprivation (PYPL)	person-years (thousand)	174	3,100	See Equation (3)
Deprived population	person-years (thousand)	228	4,634	PYAD+PYPL
Reference population	person-years (thousand)	1,217	29,549	Living population + PYPL
GD_1	%	17.9	15.7	$\frac{Deprived\ Population}{Reference\ Population}$

Figure 9: Differences in trends between GD and HC: Comoros and Botswana. HC and GD with $\hat{a} = 50$

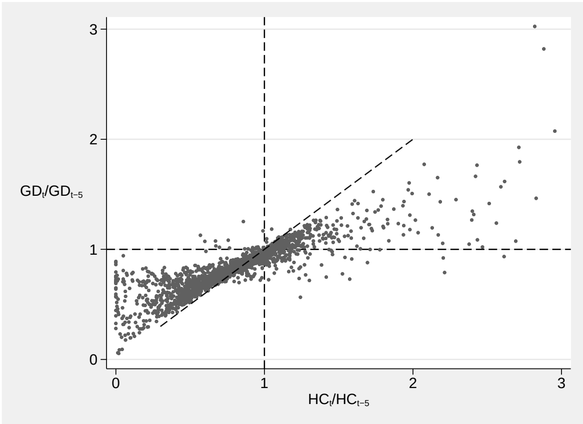


Comoros Botswana

types of deprivation remained constant while total population increased by about 50%, leading to the observed fall in GD. In the case of Botswana, HC decreased by 12% between 1990 and 2001 while GD remained roughly stable (+0.7%). This is due to the large increase in mortality rates that followed the spread of the HIV epidemics.

Comoros and Botswana are two selected examples of the opposite diagnoses one can draw when measuring deprivation using GD instead of the HC ratio. How often do these opposite diagnostics arise in the last 25 years? In Figure 10, we plot the ratio of the value of GD in year t relative to its value in t-5 for each country in our sample against that for HC. As indicated by the figure, overall, the two measures generally agree. For most countries and periods, a decrease (increase) in HC is accompanied by a decrease (increase) in GD. Note that the relation between the two measures is flatter than the 45° line, which indicates that HC varied more than GD, owing to the fact that more progress were made against alive deprivation than against lifespan deprivation between 1990 and 2015. However, the two measures do not always agree, as attested by the large number of points located in the North-West and in the South-East quadrants. These points represent 8.4% of the comparisons made: in these cases, the diagnostic of deprivation based on income contradicts the one based on total deprivation. Note that this result relies on the conservative assumption $\gamma = 1$, as this percentage goes up to 26.9% when the value of γ tends to infinity. Finally, given the increasing importance of lifespan deprivation in total deprivation, these reversals are bound to be much more frequent in the future.

Figure 10: Deprivation trends. HC and GD with $\hat{a} = 50$, t to (t-5) ratios.



Only observations with $HC_t/HC_{t-5} < 3$ are reported

6 Concluding remarks

Most measures of poverty or deprivation ignore premature mortality. In this paper, we propose two measures of "total deprivation" that combine meaningfully information on income poverty and early mortality in a population, by adding time units spent in income poverty and time units of life lost due to premature mortality. This additive approach follows from the mutually exclusive nature of the two dimensions considered, poverty and premature death. We characterize our proposed measures, show that they satisfy a number of desirable properties, and contrast their implications with existing indices. Among the two measures we propose here, the generated deprivation index is probably the most relevant, given the data available. It is based on current mortality, as measured by the number of years prematurely lost by individuals dying in the current period. It captures how much deprivation has been generated in a given year, which makes it more sensitive to contemporaneous changes in the society.

Our aggregation method allows placing an explicit and meaningful lower bound on the normative trade-off (the weight γ) between premature mortality and poverty. This lower bound is based on the view that being prematurely dead is no better than being in alive deprivation ($\gamma \geq 1$). Using this conservative approach, we estimate that about one third of total deprivation worldwide is generated by premature mortality and two third, by income poverty. Given the overall decline in income poverty worldwide, the importance of lifespan deprivation is bound to increase over time, which justifies the systematic inclusion of lifespan deprivation into deprivation measures. At the country level, ignoring premature mortality leads to biased evaluations in the level and in the evolution of deprivation.

Our indices provide a new lens through which to approach the trade-off between saving lives and the cost of doing so. To analyse such a trade-off, economists typically resort to measuring the value of life, as implicitly revealed by policy choices or legislative measures (e.g., Viscusi (1993)). This concept is however often incomprehensible in larger audiences, which prevents critical public debates from taking place. The indices we propose do not formulate this trade-off in money terms, but in the number of years spent in poverty that can be generated to save one year of life. We hope this alternative formulation will prove less controversial, thereby allowing a more peaceful deliberation.

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